# Electrical Engineering

November
1933

A membership message from President J. B. Whitehead

« See inside front cover »

Further exposition of the unified publication plan

« See page 793, news section »

Four papers that will be discussed at the forthcoming winter convention, the first issued under the newly adopted publication plan

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Published Monthly by the
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# Membership in the Institute

## ——A Message From the President

ORGANIZATION is a first prerequisite to success in any movement or enterprise. In a profession particularly, without organization there is no cohesion, no opportunity for mutual consideration and discussion and subsequent combined action in matters affecting the profession as a whole, no machinery for promoting technical development and disseminating its results, and in fact, complete absence of all combined action looking ultimately to the elevation of the profession and the welfare of the individual engineer.

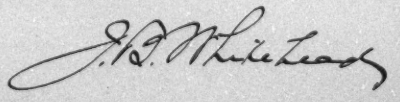
The American Institute of Electrical Engineers is an organization through which electrical engineers are united for conserving and expanding the opportunities open in the profession, for the elevation of its standards, and for the stimulation and improvement of the professional equipment of the individual member. A survey of the list of general committees of the Institute will indicate the many important directions and contacts through which the Institute keeps in touch with those social, civic, and political influences which may affect the welfare of the profession and its members. Thus the Institute concerns itself with the economic status of the engineer; questions of Institute public policy are constantly considered by a committee of experienced leaders; a continuous scrutiny of legislation affecting the engineering profession is the duty of another group; among many other matters receiving constant consideration are a code of principles of professional conduct, safety codes, the fixing of standards, and the award of prizes and medals for conspicuous achievement. The direct results of these activities are not commonly obvious, and in few cases is it possible to point out striking instances in which the welfare of the profession and its members have been protected or enhanced. Nevertheless, these general activities, representing as they do the principles, codes, and standards of the profession, constitute perhaps the most valuable functions of the Institute to the profession as a whole and so to the individual member.

The Institute, as is well known, also has as a first concern the advance of the state of the electrical art, and the cultivation and improvement of the professional qualifications of its members. Its activities in this direction are found in the conventions, technical meetings, papers, discussions, and publications. The advantages and benefits of these activities to the members have been often described as for example in Mr. Johnson's excellent article in ELECTRICAL ENGINEERING for June 1933, p. 365, and need no further emphasis at this time. It is inconceivable that any electrical engineer should willingly separate himself from participation in the benefits of the enormously valuable product of the Institute's technical programs.

What then of the prospective new member? Is it possible for any young man contemplating the electrical profession to picture to himself a successful progress or career without identification with the American Institute of Electrical Engineers, representing as it does the combined counsels of its best leaders in the interests of the profession generally, and setting forth as it does for direct service to its members, the contemporary state of the art in all its branches, in a continuous record of progress and development?

And what should be the attitude of an Institute member to the Institute itself? Should not his foremost question be, "what can I do to aid the work of the Institute, what can I contribute to the high standards it sets of professional conduct, attainment, and progressive development?" Should not his question be always, "what can I do for the Institute?" rather than, "what does the Institute do for me?" For every member must realize that ultimately his own professional and material welfare is intimately bound up with that of the Institute. The work of the Institute cannot be curtailed, its resources cannot be impaired, without ultimate detriment to the professional and material well-being of each of its members.

"If a man deserves to have the advantage of living in an organized community, he has to consult not only his own fortune, but also that of the society, * * * * He must realize that its prosperity is his own prosperity, and that it cannot suffer without his own injury." (Haeckel.)



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**Officers and Committees** (For complete listing see p. 658–81, September 1933 issue of ELECTRICAL ENGINEERING)

★ THESE PAPERS are to be discussed at the forthcoming A.I.E.E. 1934 winter convention to be held in New York City, January 23–26, 1934.

BOARD of directors of the A.I.E.E. acted upon many matters of importance to members of the Institute at its meeting of October 20. Several by-laws were amended, several Institute representatives were appointed, and several important resolutions were adopted. *p. 794–5*

ACTIVITIES of 5 A.I.E.E. technical committees during 1932–33, and advances in their respective fields, are reported in this issue. *p. 746–7; 757–8; 766–70*

PRELIMINARY standards for noise measurements approved June 26, 1933, by the sectional committee on acoustical measurements of the American Standards Association, provide a universal language for sound measurements. Comments and criticisms of these standards are solicited. *p. 741–6*

SEVERAL factors affect the reliability and efficiency of steam turbines, and several methods have been developed for maintaining efficiencies. *p. 748–51*

PLANS for the A.I.E.E. 1934 winter convention already are progressing rapidly; a general committee has been appointed, 12 technical sessions have been arranged, and other events scheduled. *p. 795*

ENGINEERING education is not immutable, but neither is it drifting—so speaks Dean Sackett of the Pennsylvania State College in commenting on the present status of engineering education. *p. 796*

A SIMPLIFIED method of calculating circuits, called the "short-circuit current solution," is described in this issue. It is believed to be new, and is applicable to both a-c and d-c circuits. *p. 776–9*

TROLLEY wire lubrication, early recognized as an important part of electric railroad operation, is the subject of extensive improvement resulting from an entirely new and different method of applying a solid lubricant. *p. 771–6*

THEORY of probability found its first impetus more than 400 years ago in the questionable motives of a notoriously superstitious and irrational group—one of inveterate gamblers. Today the theory has many applications of a practical and scientific nature. *p. 752–7*

PREPARATORY to the transmission of electric power from Boulder Dam to the system of the Southern California Edison Company, several stability studies were made. One important finding of those studies was that by inserting a resistance in series with generator armature windings, a tremendous gain in power limit was obtained. *p. 758–66*

METHODS for determining the steady state stability of electric power systems have been developed that are independent of machine inertias. The first of 2 papers on this subject presents methods for determining the steady state power limits of systems composed of shunt impedance loads and ideal cylindrical-rotor synchronous machines, *p. 780–7*. The second paper generalizes the methods presented in the first and applies them to induction machines, composite loads, and interconnected systems. *p. 787–92*

# The Engineer in Public Utilities Regulation

By A. V. GUILLOU
MEMBER A.I.E.E.

Chief Engineer, Public Service Commission of Wisconsin

**Many years ago the need for public regulation of public utilities was recognized. As these natural monopolies increased in number and in scope, regulatory powers, which at first were exercised by legislative bodies, gradually were delegated to specially appointed commissions until now there are about 50 such bodies in the United States. Engineers employed by these commissions should have not only a technical education, but also a knowledge of construction, operating, accounting, and commercial problems and procedures.**

WHILE the regulation of some types of public utilities has come very much into the public eye during the past few years, the fundamental principles involved are by no means new. A great many years ago when England was a comparatively sparsely settled country the traveler by horseback or stagecoach would have found himself in a very unfortunate position had he been unable to secure accommodations at the single inn at the end of his day's journey. The law of that time recognized this situation; the innkeeper was required to extend his accommodations upon an equal basis to all who came to his door, and his charges were regulated. Side by side with the tradition that the Englishman's house was his castle grew up the principle that some businesses were affected by a public interest and subject to regulation by the public.

An interesting commentary upon the progress since that time is that the businesses which then were considered properly subject to public regulation no longer are regulated, and that the businesses which now are regulated were unknown then and probably undreamed of. With the exception of water, the commodities and services provided by present day public utilities were unknown in those days.

We now live and travel in and between cities instead of settlements. If we do not like the accommodations offered by the present day innkeeper we easily travel farther between dinner and bedtime than our forefathers could in a long hard day. The innkeeper no longer has a natural monopoly on the business of the wayfarer; competition, instead of governmental regulation, now controls his prices and accommodations. Our congested cities are vitally dependent upon modern utility services, such as water, gas, electricity, communication, and transportation. These businesses are natural monopolies, and regulation has come to control their rates and services. There are those who see in the present problems in the transportation field the coming of competition as a successor to regulation; and it is probable that the developments of the past hundred years are continuing slowly around us.

## Early Regulation in the United States

These changes in our community life have been accompanied by changes in the organization of business enterprises that have brought corresponding changes in the methods as well as in the field of regulation. During the first decades of their existence, some of the New England states enacted laws governing the conduct of toll roads or bridges. With the development of the railroad the problems of regulations were complicated greatly by the growth of large corporations, extensive systems, and the technical details involved in their operation. Regulatory powers formerly exercised by the legislative bodies were delegated to commissions that could bring to the solution of these problems intensive study and the benefits of a continuity of experience. With the development of the gas and electric utilities, regulatory powers at first were exercised by the legislative bodies, usually the city councils; there has been a continuous delegation of that authority to state commissions.

As the country built up and railroad systems extended across state after state, the commissions of the individual states began to meet the problems and limitations inherent in the exercise of their jurisdiction over only part of large and unified operations. As a result federal regulation of railroads was established through the agency of the Interstate Commerce Commission. Electric transmission lines and natural gas pipe lines are assuming interstate characteristics, and it is not unnatural to find a growing demand for nation-wide regulation of these industries.

## Powers of State Commissions

While the powers of these state commissions vary between widely separated limits in the 47 states in which they are found, in general they are related directly to the duties which the commissions are expected to perform. The primary object of these commissions is to see that the public shall receive adequate service at reasonable rates. Most of them therefore are authorized to require the utility companies under their respective jurisdictions to provide the proper facilities for maintaining adequate service.

Adequate service and reasonable rates cannot be given by a company staggering under a burden of

Written especially for Electrical Engineering; based upon a paper presented at a joint meeting of the Madison (Wis.) Section and the University of Wisconsin Branch of the A.I.E.E., April 12, 1933. *Not published in pamphlet form.*

carrying charges on excessive debts or security issues. Many of the commissions, therefore, have been given control over the amount of securities that may be issued by the utility companies and the prices at which they may be sold. Competition often leads to wasteful duplication and companies subject to competitive attacks cannot operate as satisfactorily or economically as otherwise would be possible. Hence, many commissions have been given control over competition, and utilities must secure authorization before entering new fields. The commissions must have adequate information upon which to base their orders and decisions; many of them, accordingly, have been given control over methods of accounting and the filing of reports by the utilities under their jurisdictions. Of the same nature is the authority to make valuations of property and other special investigations for the development of necessary information. All of these powers lead up to the ultimate purpose of controlling rates. Rates charged by utilities generally are required to be uniform and nondiscriminatory, to be kept on file for public inspection, and to be subject to change by the regulatory commission.

The present day institution of government regulation of utilities is probably much more the result of development than of design, and the development is continuing. Generally speaking, the powers of state and federal commissions have been increasing gradually, each specific power or duty usually being traceable to the recognition of a need. As these needs continue to arise, the corresponding lines of authority doubtless will continue to extend. The holding company has developed largely during the past 10 years, and within the past 5 years there has been a very noticeable extension of commission jurisdiction over the relations between operating and holding companies.

## Many Decisions Appealed to Federal Courts

Early in the history of regulation came appeals to the courts to enjoin the alleged arbitrary exercise of authority which had been delegated to the early commissions. Many commission decisions have been appealed to the federal courts as being in violation of the fifth and fourteenth amendments to the Constitution of the United States. It is interesting to note that the fifth amendment was ratified shortly after the adoption of the Constitution as part of a bill of rights designed to protect our forefathers against an abuse of power by the central government which they had just set up. In addition to forbidding the taking of property without due process of law or the taking of private property for public use without just compensation, that amendment protects the defendant against being compelled to testify against himself or being placed twice in jeopardy for the same offense. The fourteenth amendment, ratified just after the Civil War, partly for the protection of the recently emancipated slaves, prohibits the states from depriving any person of life, liberty, or property without due process of law, or denying to any person the equal protection of the law.

Before a court a utility corporation becomes a person, entitled to the equal protection of the laws, and whose property may not be taken for public use without just compensation. Out of many decisions in which the courts have defined and interpreted these rights have come certain generally accepted principles and some which are still in a process of development. It is recognized generally that the property which cannot be taken without due process of law, or for public use without just compensation, includes the use of a utility's system as well as the system itself. Just compensation for the use of utility property has been stated to be a fair return upon the value of that which is employed for the public convenience, but the exact definition of the term "value" is still a subject of debate. Those who view the matter from an economic background have not given up hope that actual or prudent investment may be controlling, while the courts have held that dominant consideration must be given to the cost of reproduction at current prices.

The idea of "cost of reproduction" is far simpler in the abstract than in its detailed application, particularly when coupled with the mandate that it is the property in question that is to be valued, rather than some substitute for it. For example, by giving dominant consideration to the cost of reproduction and observed depreciation, the reader may determine the present value of a model $T$ Ford automobile, remembering that model $T$ is no longer in production, that model $A$ is quite a different car, and that the easily ascertainable price of model $A$ is a forbidden answer under the "substitute plant" theory. The obvious check against the opinion of a used car dealer cannot be applied to earlier types of generating equipment, meters, etc., and this crude example only illustrates some of the questions involved in the use of "cost of reproduction" that have helped to make valuation work a special field of engineering.

Having determined, by due process of law the value of the property in question, the regulatory commission decides upon a return equal to that generally being made at the same time and in the same general part of the country on investments in other business undertakings which are attended by corresponding risks and uncertainties. The utility is entitled to rates that will permit it to earn such a return, after meeting proper operating expenses and depreciation requirements, although at times competitive or other conditions may dictate the application of lower rates. Representatives of the parties to a rate case would be more than human if their claims and figures were not at least slightly colored by their own interests; consequently, complete agreement seldom is reached upon value, the propriety or necessity of operating expenses, depreciation accruals, etc. It is upon such questions that the technical staff of the commission is called upon for independent reports or the analysis of evidence presented.

Among approximately 50 commissions in the United States that have power to regulate utilities of various kinds and in various localities, many

types of organization will be found. The Oregon commission consists of 1 man; others include various numbers of members up to 11 that comprise the Interstate Commerce Commission. As these men are faced by problems of an extremely wide variety, they are assisted by staffs of attorneys, accountants, engineers, and others, varying in number from a half dozen in some states to well over a hundred in others.

### Duties of Commission Engineers

Engineers employed by the state commissions are called upon for the preparation of complete detailed valuations and appraisals of small and large properties; for studies of depreciation and operating expenses; to inspect physical facilities and recommend changes necessary for safety to the public or employees, or for improvement in service; and for extensive work in the examination or preparation of rate schedules and the promulgation of rules governing the quality of service or business relations between the companies and their customers. Qualifications of men who can do or direct such work are not easy to fill. To meet these problems properly the engineer must understand the design of the equipment which he is appraising or inspecting, yet his duties seldom include any actual design work. He must be familiar with the methods of operation of these properties, yet neither he nor the commission by which he is employed actually operates any utility. He should understand the legal principles governing his work, and be familiar with the financing of the companies with which he comes in contact and with the effect which financial requirements may and should have upon their operations. A large part of his basic information comes from the accounting systems and accounting reports of the utilities; hence, he must have sufficient practical knowledge of accounting to know how to get the figures he needs and to appraise their reliability and practical accuracy. He is called upon frequently to settle minor complaints of customers regarding bills, deposits, etc.; thus in addition to technical knowledge, he must have an understanding of business methods and relationships.

The commission engineer often must present the results of his work to attorneys and executives, or from the witness stand where he is subject to cross examination. The ability to explain technical matters clearly in non-technical language and to discuss controversial questions with those who are in an antagonistic frame of mind is more important than in many other fields of engineering work. Regulatory work offers to the qualified engineer exceptional opportunities for broad experience and contacts with men high in the profession and industry. To make the most of these opportunities, the engineer should take to his work with a regulatory commission not only a technical education, but also an understanding of construction, operating, accounting, and commercial problems and methods that seldom is obtained except through several years of varied experience in the organization of an operating utility.

# Production and Utilization of Micro-Rays

**What is said to be the first commercial application of radiotelephone communication on ultra short wave lengths is to be placed in operation between Lympne, England, and St. Inglevert, France. Some facts concerning the methods of producing these oscillations are presented herewith.***

NO DOUBT many will recollect statements that appeared in the press after the successful demonstration between Dover, England, and Calais, France, on March 31, 1931, of a duplex radiotelephone link operating on a wave length of about 18 cm. Since that time an order has been placed for a permanent link between Lympne, England, and St. Inglevert, France. This will be the first commercial application of communication by these extremely short waves. It is believed, therefore, that an article dealing with the method used to produce oscillations of a wave length shorter than 20 cm will be of immediate interest. These waves are known as "micro-rays."

The tube used contains 3 electrodes having a cylindrical symmetry (see Fig. 1). The tungsten filament is on the axis of the structure. A helicoidal electrode centered on the filament is called the "oscillating electrode." The external cylindrical electrode is called the "reflecting electrode." The 2 extremities of the oscillating electrode are connected to a transmission line which leads to a radiating element. The simplest case is that in which the transmission line is of the parallel conductor type and the radiating element a small length of wire at right angles to the transmission line.

When the system is adjusted correctly there is a current maximum in the radiating element. This element remains small compared with the wave length and functions as a theoretical "doublet." That is to say, the amplitude of the high frequency current is substantially the same in all points of this doublet at any particular time. When the transmission line is short, the radiation of the doublet is predominant and all other radiation effects in the system may be considered as secondary.

A positive potential of about 250 volts is applied to the oscillating electrode by means of a connection perpendicular to the doublet at its middle point. The reflecting electrode voltage is negative with respect to the filament.

Oscillations are produced in the part of the static

* Abstracted from an article "Production and Utilization of Micro-Rays" by A. G. Clavier, Les Laboratoires, Le Matériel Téléphonique, published in *Electrical Communication*, July 1933. Abstract prepared by Frederic C. Young (M'30) development engineer Stromberg-Carlson Telephone Manufacturing Company, Rochester, N. Y. *Not published in pamphlet form.*

characteristic corresponding to voltage saturation of the filament by the d-c voltage of the oscillating electrode. Electrons are attracted by the positive oscillating electrode. Some electrons fall on the oscillating electrode; others travel through the meshes of this electrode, are retarded, and finally turn back toward the oscillating electrode; some of these fall on this electrode while others reach the filament region and are not distinguishable from the electrons initially leaving the filament.

When a high frequency voltage appears along the oscillating electrode, the total number of electrons that fall at any point will not always be the same; and at any point of the oscillating electrode there will be a leakage current made up of an average d-c value and a high frequency component which will not be in phase with the original high frequency voltage along the electrode. This dephased leakage current makes it possible to sustain the ultra short wave oscillations within the tube.

### Constant Frequency Curves

It was found that for a certain external circuit, there is one particular frequency for which the tube gives its maximum output. This frequency is found by varying the tube voltages until maximum output is obtained. The frequency at which this occurs is the desired optimum. For this frequency the constant frequency curve is plotted. This curve gives the necessary relation between the voltages on the reflecting and oscillating electrodes, to produce the optimum frequency.

The constant frequency curves are very important for the correct utilization of the micro-ray tube. For instance, they give the possibility of obtaining amplitude modulation. In Fig. 2, the right-hand side of the output curve shows that the output there varies almost linearly in a certain region provided the relation between $E_o$ (potential of oscillating electrode) and $E_r$ (potential of reflecting electrode) follows the law indicated by the constant frequency (dotted) curve. Now this law gives also a linear relation between the 2 voltages. It is thus possible to apply the modulation voltage on both electrodes in such a way as to keep the correct relation between the voltages applied on the tube, and consequently the same frequency.

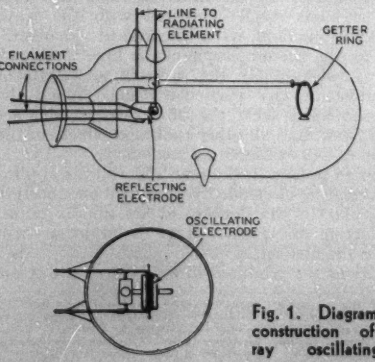


**Fig. 1. Diagram showing construction of micro-ray oscillating tube**

### Influence of External Circuit

The power which the electronic process inside the tube makes available is dissipated in the external circuit comprising, as already explained, a short transmission line leading to a radiating doublet.

Oscillations are sustained provided the apparent resistance as viewed from the tube does not exceed the negative resistance produced by the difference in phase between the leakage current and the high frequency voltage across the oscillating electrode. Supposing this condition to be fulfilled, the optimum frequency will be obtained when the oscillatory circuit is resonant on that frequency for which the electronic phenomena inside the tube are adjusted.

For the Dover-Calais experiment in 1931 the radiating element or doublet was placed at the focus of a paraboloidal reflector. It has been found more convenient in certain cases to locate the transmitting or receiving tube behind this reflector in order to control or change the tubes more easily. This can be done by means of a transmission line connecting the oscillating circuit to the radiating system, which can take the shape of a half wavelength antenna.

As previous experiments have shown that replacing the doublet, which is of low resistance, by a half-wave antenna did not give quite as good results, it has been found necessary to step down the resistance offered by the half-wave antenna; this may be done with a quarter wavelength transmission line, the characteristic impedance of which is properly chosen. It is also essential to avoid all undue losses, especially radiation losses in the tube oscillating circuit.

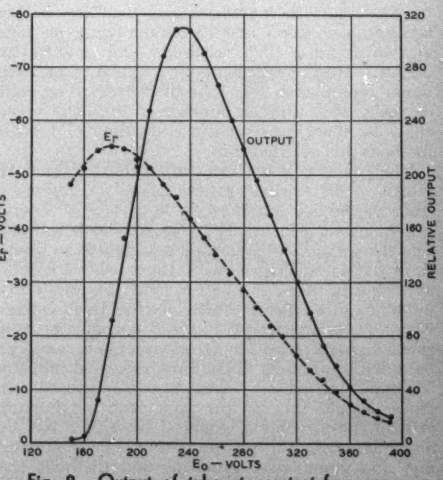


**Fig. 2. Output of tube at constant frequency**

Dotted curve shows relation between $E_r$ and $E_o$ for constant frequency

# Noise Measurement Being Standardized

**In the following 2 articles, recent progress in noise measurement and its standardization is outlined. In the first, P. L. Alger, chairman of the A.I.E.E. committee on sound, points out the practical significance of preliminary standards for noise measurements approved June 26, 1933, by the American Standards Association, and indicates the lines of further progress required to bring noise measurements into general use. In the second article, Harvey Fletcher, chairman of the A.S.A. sectional committee on acoustical measurements, presents the formal report of that committee which contains the proposed standards and a résumé of pertinent discussion thereon. These standards provide a universal language for sound measurements in general, and a foundation for future standards of noise meter calibration and apparatus noise measurements. Members of the Institute are urged to review these new possibilities in sound measurement and to put them into practical use. Comments and criticisms of the proposed standards are solicited by the committees.**

## I—Progress in Noise Measurements

By
P. L. ALGER*
FELLOW A.I.E.E.

General Elec. Co.,
Schenectady, N. Y.

THE INITIAL STANDARDS of acoustic measurements, presented by Doctor Fletcher in the article following this one, form a milestone of progress which deserves attention. They create a universal language in which distant observers can communicate and preserve the results of their investigations. They permit engineers to evaluate numerically the noise performance of machines of varied designs. Ultimately they will give to the public a means for translating its now vague desire for quiet into concrete legal and commercial terms.

Work on these standards began in January 1932, with the organization of the sectional committee on acoustical measurements of the American Standards Association, with a large membership representing engineering and scientific societies, trade associations, and the government. The subcommittees immediately responsible for the work include representatives of the major acoustical laboratories of the country.

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* Chairman, A.I.E.E. committee on sound.

The standards define the essential elements in the measurement of pure tones in free air waves, and specify means for determining the loudness of complex tones through averaging the judgments of a large number of observers. They provide immediately available means of making laboratory sound measurements, and a foundation for future standards of noise meter calibration and apparatus noise measurement.

In this article, salient features of the standards will be commented on, the development of total-noise meters will be discussed, and possible means for apparatus noise measurement outlined.

### PRELIMINARY A.S.A. STANDARDS

The standards establish a measurement scale for a standard reference tone, selected to be easily reproducible and in the range of maximum audibility; and they provide for measuring all other sounds on the same scale by determining the equally loud reference tone. There are 5 salient features of these standards.

First, a pure tone of 1,000 cycles per second frequency is established as the reference for loudness comparisons. This coincides with the agreement reached at the A.I.E.E. Middle Eastern District meeting held at Rochester, N. Y., in May 1931, which has been in general use in the United States since.

Second, the reference, or zero, level of reference tone intensity is established as $10^{-16}$ watts per square centimeter, corresponding to an rms pressure of 0.000207 bar (a bar is a pressure of one dyne per square centimeter) in a free air wave under standard atmospheric conditions. In the Rochester agree-

ments, a pressure of 0.001 bar was adopted as the reference level, so that the new level is 13.8 decibels (db) lower than the old. On the new scale, negative sound intensities will be avoided because the zero level closely approximates the normal threshold of hearing for 1,000-cycle tones.

Third, the intensity level of the reference tone is specified as the number of decibels above the reference level. Therefore, if $I_1$ is the energy flow of the reference tone in watts per square centimeter, and $I_0$ is the reference level of $10^{-16}$ watts per square centimeter, the intensity of the tone in decibels is $10 \log_{10} (I_1/I_0)$, or simply $10 \log_{10} (I_1 \times 10^{-16})$. This logarithmic or decibel scale is the same as that generally used during the past few years, except for the lower zero of the new scale. The immense energy range of audible sounds of roughly 3 million fold thus is compressed into a convenient numerical scale of about 125 db. On a decibel scale, each time the energy of a wave is halved, the intensity is reduced only 3.01 db. Thus 2 identical waves occurring together have an intensity 3 db greater than either alone. It is necessary to decrease the sound energy to 1 per cent or the sound pressure to 10 per cent of its initial value to secure a reduction of 20 db. Table I gives approximate values of typical sound intensities on the old and new scales.

**Table I—Approximate Sound Intensities in Decibels**

| | Old Scale Reference Level 0.001 Bar | New Scale Reference Level $10^{-16}$ Watts Per Sq Cm |
|---|---|---|
| Threshold of hearing | −13.8 | 0 |
| Very quiet bedroom | +15 | 29 |
| Ordinary home | 25 | 39 |
| Ordinary office | 35 | 49 |
| Machine shop | 60 | 74 |
| Limit of conversation | 75 | 89 |
| Threshold of feeling | 110 | 124 |

Since in practical cases sound wave reflections create interference patterns, the actual values of sound pressure, particle velocity, and intensity or energy flow at any point, have varying relations different from those existing in plane or spherical sound waves. The response of the ear depends in some degree on all 3 factors, the precise relations being unknown. The entry of the observer's head into the field in fact modifies the sound pattern, making it difficult to define the relationship between the pick-up of any microphone and the response of the ear. A scale of sound pressure measurements to supplement the intensity scale, therefore, is included in the standards, but is not recognized as a direct measure of loudness. As an alternative method of expressing loudness, for use whenever a greater numerical range than that given by the decibel scale seems desirable, the report tentatively suggests that values be taken from Fig. 2, which represents the average of observers' judgments on a linear scale of loudness (see "Loudness, Its Definition, Measurement and Calculation," by H. Fletcher and W. A. Munson, to be published in *Journal* of the Acoustical Society of America).

Fourth, the loudness of a pure tone is defined tentatively by a new set of constant loudness curves recently determined by the Bell Telephone Laboratories, (Fig. 1 of report). These curves cover a greater frequency range than Kingsbury's chart, heretofore generally used. The new curves give high intensity tones of less than 100-cycle frequency, about 5 db greater loudness than the old, and they also differ materially at frequencies above 2,000 cycles.

Fifth, the loudness of any other sound is defined as the intensity of the equally loud reference tone, as determined by the average judgment of an indefinitely large number of observers. The standards require actual judgments by a large number of observers comparing the test sound with a 1,000-cycle reference tone as the only means of measuring a complex noise. This constitutes a laboratory standard of reference, therefore, to be supplemented ultimately by working standards based upon some agreed method of combining the different frequencies and intensities existing in complex tones.

## Total-Noise Meters

Practical use of the noise measurement standards requires the development of sound meters that will measure actual sounds in accordance with the established scales. For pure tones, this is relatively easy, all that is required being a calibrated microphone, an amplifier, a frequency weighting network in accordance with Fig. 1, and a suitable ammeter. While there are difficult technical problems in microphone calibration, which may require additional standards, the whole problem is clear cut so long as pure tones are considered.

For complex tones, however, it is necessary to design the meter so that it will combine the component frequencies and intensities in the same manner as the ear of the average observer. Usual meters record the rms value of the different frequencies present, but the ear adds them in a more complex way giving less weight to closely adjacent frequency components. Fletcher and Munson have proposed a system for analysis and subsequent synthesis of complex tones to determine the true loudness, but this method is time consuming and as yet fraught with uncertainties.

The immediate program before the standards committees, therefore, is to develop auxiliary standards for total-noise meters that will assure like results being obtained by all laboratories. The first step in this work probably will be the circulation of standard noise sources among the interested laboratories; these laboratories will measure the loudness of the standard noises under comparable conditions, and subsequently analyze and correct for discrepancies between their results.

In developing these meter standards, it is desirable to require calibration for complex noises in accordance with the average of a number of observers' judgments; but the recognized difficulty in making a single meter imitate the ear for all tone combina-

tions may force preliminary standardization on one or a series of weighting networks. Calibrations of noise meter readings for each broad type of complex noise, such as fans, street noises, etc., to bring them in accord with the ear, may then be developed as need arises. The constant loudness curves of Fig. 1 of the present standards logically may be employed for weighting networks in these preliminary meter standards. The 60-db level seems most suitable for general noise measurements, while additional networks matching the 30- and 90-db levels are adapted for special low and high intensity work, respectively.

## Apparatus Noise Measurements

The correct test of the noise produced by any piece of apparatus is to measure the noise level at the actual locations involved, after the apparatus is installed and while it is operating under normal conditions. However, just as in lighting problems, the intensity of illumination obtained is affected tremendously by the condition of the walls, distance from the source of light, and obstructions that may be in the way of direct light beams; so in acoustics, the sound level depends on the environment almost as much as on the noise energy leaving the source.

To ascertain the "nuisance" value of the noise produced by any piece of apparatus, it is accordingly necessary to go much further than simply to make sound measurements at a single point in space near the apparatus. For each type of noise source, a suitable distance and direction from the source to the point of measurement must be selected, and allowances then must be made for the acoustic reflections and absorptions due to the environment. As compared with light, the coefficients of acoustic reflection are generally higher and less controllable, while our inability to observe directly the sound patterns in a room makes it much more difficult to avoid gross errors from shadows. Also, the comparatively long wave lengths of sound, and the wide range of frequencies involved, make the shadows, or sound patterns, very markedly for the different frequency components of complex sounds. In many cases, therefore, the total sound emission from a source must be measured by integrating readings taken at all angles, just as is done in the determination of mean spherical candlepower.

Representative noise values for small pieces of apparatus can be obtained in a large room with sound absorbing walls, since the wall reflections then are negligible at short distances from the source. As the apparatus becomes larger, the room smaller, and the walls more reflecting, the sound patterns become more important, and often the variations from this cause will far exceed variations in the individual pieces of apparatus tested. It may be possible to specify the reverberation time of the room (time for a given sound to die away to one millionth of its initial energy as a result of absorption from repeated wall reflections) and the locations of the sound pick-up and of the source with respect to the walls, so as to permit standardized noise measurements; or, it may be preferable to revolve the microphone or swing it on a pendulum to secure an average reading. Revolving sound reflectors, and even highly reflecting walls may be best in other cases, to amplify weak sounds or to average out irregularities in the sound pattern. All of these conditions that affect the sound intensity and pattern must be described explicitly in any report of noise test results if the observations are to have any meaning. To give noise values without these explanatory data is as absurd as to report the room illumination from a given lamp without specifying whether or not a reflector was used, and whether the ceiling was white or dark.

Finally, the level of room noise must be lower, preferably by at least 10 db than the noise of the apparatus to be measured. Noise measurements made in an open factory are as unsatisfactory as reading the candlepower of a lamp in open daylight.

Difficulties involved in getting a single reading that will represent adequately the sound emission of a source must not be underestimated. If one can imagine measuring the total light emission from a large luminous chameleon, whose color distribution is fluctuating violently, some impression will be obtained of the problem of measuring the noise of an electric motor. In the present state of the art, therefore, only complete sound analyses made and interpreted by specialists can be relied upon in evaluating the comparative noise production of apparatus.

More important than measurement of the airborne noise in many cases is the determination of the noise resulting from vibration transmitted through the apparatus foundation. For example, an extremely small motion of the feet of an electric motor, which by itself is quite inaudible, may be communicated to a large area, such as the ceiling of the room below, and so may produce disproportionately large amounts of noise. Furthermore, such vibration may be transmitted through several walls and floors of a building, reappearing at a point where the existence of some resonance may produce appreciable amplitudes and correspondingly large noises (see "Elastic Supports for Isolating Rotating Machinery," by E. H. Hull and W. C. Stewart, A.I.E.E. Trans., v. 50, 1931, p. 1063–8). It is highly important, therefore, to limit transmitted vibration, and to define the performance of apparatus in this respect quite aisde from its production of air-borne noise.

The most practical way to do this seems to be to measure the noise produced by a flat slab forming the ceiling of a box or small room, when the apparatus in question is mounted on top of it, and all other sources of noise are excluded.

## Summary

The work on standards so far accomplished provides a universally acceptable language for describing acoustic measurements. The general adoption of this language will stimulate greatly the making, reporting, and discussion of noise tests, and thus will bring progress in noise reduction. It seems opportune now for specialists in the various fields of noise

measurement to come forward with papers and discussions, presenting the results of their use of these new standards and suggesting practical rules for making comparative noise tests of all sorts. Sure progress will result from the publication of such material, and its analysis ultimately will enable additional standards of noise measurements to be agreed upon that will serve the needs of particular industries. [EDITOR'S NOTE: In this connection, an article on comparisons of sound meter measurements with observers' judgments of loudness, by P. H. Geiger and E. J. Abbott, already is scheduled for publication in some future issue of ELECTRICAL ENGINEERING.]

The field of noise measurements offers an excellent opportunity for universities to combine educational and research work with public service. The educational process is stimulated greatly by putting into the students' hands equipment for exploring unknown fields. We live immersed in an ocean of sounds, which heretofore have remained invisible and uncontrolled, although they affect our lives in innumerable ways. When the new meters are available, we shall be able to measure these sounds, analyze them, and control them through specifying and paying for exactly the quietness level we desire. To develop the use and appreciation of these measuring devices is a scientific job requiring enthusiastic experimentation, which no one is better fitted to carry on than university students and research staffs.

In the immediate future, the institute committee on sound will concentrate its attention upon securing agreement on standards for meter calibrations, and the use of weighting factor networks. Meanwhile, members of the Institute are urged to review these new possibilities in sound measurement and to put them into practical use.

# II—Proposed Standards for Noise Measurements

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THE PROPOSED STANDARDS and definitions given in this article were approved on June 26, 1933 by the sectional committee on acoustical measurements of the American Standards Association. They now are being circulated widely to secure criticism and suggestion and will be reconsidered in light of the comments received. It is for this reason that they were presented before the A.I.E.E. at the 1933 summer convention held in Chicago, Ill., and are being published now in ELECTRICAL ENGINEERING.

These standards define numerical scales and other essentials for measuring the loudness and intensity of sounds. A reference tone is selected together with a numerical scale for defining its magnitude. The magnitude from a loudness standpoint of any other sound is measured on the same scale in terms of the equally loud reference tone.

Besides the items formally approved by the sectional committee, a discussion of these tentative standards which formed a part of the subcommittee's report also is given.

## A.S.A. SECTIONAL COMMITTEE REPORT

In order to make clear the particular meaning of intensity as used in this report the following definition taken from the report of the A.S.A. subcommittee on terminology is here given:

*Sound Intensity* (I). The sound intensity of a sound field in a specified direction at a point is the sound energy transmitted per unit of time in the specified direction through a unit area normal to this direction at the point. The unit is the erg per second per square centimeter but may also be expressed in watts per square centimeter.

Note a. The sound intensity in any specified direction $a$ is given by the equation

$$I_a = \frac{1}{T}\int_0^T p\, v_a\, dt$$

where $T$ is the period, $p$ the instantaneous sound pressure, and $v_a$ the component in the specified direction $a$ of the instantaneous particle velocity.

Note b. In the case of a plane or spherical free progressive wave having the effective sound pressure $P$ (bars) the velocity of propagation $c$ (centimeters per second) in a medium of density $\rho$ (grams per cubic centimeter) the intensity in the direction of propagation is given by

$$I = \frac{P^2}{\rho c} \text{ ergs per sec per sq cm}$$

This same relation often can be used in practice with sufficient accuracy to calculate the intensity at a point near the source with only a pressure measurement. In more complicated sound fields the results given by this relation may differ greatly from the actual intensity.

When dealing with a plane or a spherical progressive wave it will be understood that the intensity is taken in the direction of propagation of the wave.

## PROPOSED STANDARDS

1. The reference intensity for intensity level comparisons shall be $10^{-16}$ watts per square centimeter. In a plane or spherical progressive sound wave in air, this intensity corresponds to an rms pressure $p$ given by the formula

$$p = 0.000207\sqrt{\frac{H}{76}}\sqrt{\frac{273}{T}}$$

where $p$ is expressed in bars (dynes per square centimeter) $H$ is the height of the barometer in centimeters, and $T$ is the absolute temperature. At a temperature of 20 deg C and a pressure of 76 cm of mercury, $p = 0.000204$ bars.

Full text of a report of the sectional committee on acoustical measurements of the American Standards Association. *Not published in pamphlet form.*

*A.S.A. subcommittee on noise measurement: Harvey Fletcher, *chairman*, Bell Tel. Labs., New York, N. Y.; V. L. Chrisler, U. S. Bureau of Standards, Washington, D. C.; E. E. Free, New York, N. Y.; C. R. Hanna, Westinghouse Elec. & Mfg. Co., E. Pittsburgh, Pa.; H. B. Marvin, Gen. Elec. Co., Schenectady, N. Y.; R. F. Norris, C. N. Burgess Labs., Madison, Wis.; F. A. Firestone, Univ. of Mich., Ann Arbor; J. S. Parkinson, Johns Manville Corp., New York, N. Y.; J. C. Steinberg, Bell Tel. Labs., New York, N. Y.; W. Waterfall, secy., Acoustical Soc. of Am., Chicago, Ill.; S. K. Wolf, Elec. Research Products Corp., New York, N. Y.; R. G. McCurdy, Am. Tel. & Tel. Co., New York, N. Y.; and E. D. Cook, RCA Victor Co., Camden, N. J.

2. The intensity level of a sound is the number of decibels (db) above the reference level.

3. The pressure level of a sound is 20 times the logarithm to the base 10 of the ratio of the pressure $p$ to the reference pressure $p_0$. The unit of pressure level is the decibel.

4. The reference pressure $p_0$ for sound pressure measurements is 0.0002 bars.

5. A plane or spherical souud wave having only a single frequency of 1,000 cycles per second shall be used as the reference for loudness comparisons.

Note: One practical way to obtain a plane or spherical wave is to use a small source, and to have the head of the observer at least one meter distant from the source with the external conditions such that reflected waves are negligible as compared with the original wave at the head of the observer.

6. The loudness level of any sound shall be the intensity level of the equally loud reference tone at the position where the listener's head is to be placed.

7. In observing the loudness of the reference sound, the observer shall face the source, which should be small, and listen with both ears at a position so that the distance from the source to a line joining the 2 ears is one meter.

Note: The value of the intensity level of the equally loud reference sound depends upon the manner of listening to the unknown sound and also to the standard of reference. The manner of listening to the unknown sound may be considered as part of the characteristics of that sound. The manner of listening to the reference sound is as specified.

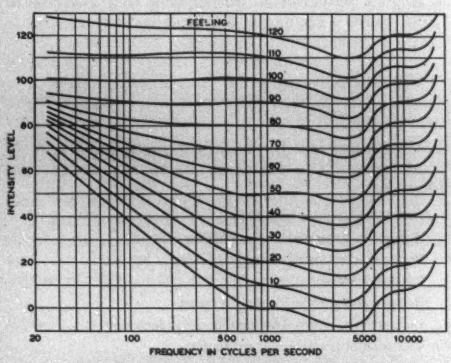
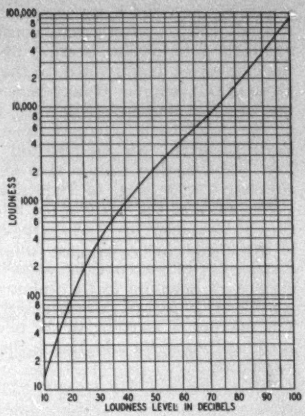



**Fig. 1. Curves of constant loudness for defining the loudness level of a pure tone; intensity levels given in decibels**

## Additional Tentative Standards for Noise Measurements

For the purpose of practical noise measurements and the design of sound or noise meters there is need for agreement upon equal loudness contours of pure tones. Also, there is some need for an additional loudness scale based upon a group judgment as to what constitutes a certain percentage reduction in the loudness of sound. For both of these things reliance necessarily must be placed on experimental data. Several investigations have been made for the purpose of obtaining such data; from these data can be derived relations that can be taken as the best indications of the present material, with, however, the necessary recognition that they are tentative and may need modification as further data are obtained.

**Fig. 2. Relation between loudness and loudness level**




8. The loudness level of a pure tone propagated as a plane or spherical wave in air, and having a frequency of $f$ cycles per second and an intensity level of $\beta$ db shall be defined by the set of curves given in Fig. 1.

9. Until more accurate data are available, the relation between loudness and loudness level shall be that given by the curve shown in Fig. 2.

## Discussion

There was considerable discussion in the committee as to what should be chosen for the reference or zero level. In many ways the threshold of hearing intensity for a 1,000-cycle tone seemed like a logical choice. However, variations in this threshold intensity arise depending upon the individual, his age, the manner of listening, the method of presenting the tone to the listener, etc. For this reason, no attempt was made to choose the reference intensity as equal to the average threshold of a given group listening in a given way. The reference intensity given in recommendation 1 was chosen because it was a simple number that was convenient as a reference for computation work, and also because it is in the range of threshold measurements obtained when listening in the manner outlined under recommendation 7. This reference intensity corresponds to the threshold intensity of an observer who might be called a reference observer. An examination of a large series of measurements on the threshold of hearing indicates that such a reference observer has a hearing that is slightly more acute than the average of a large group. For those who have been thinking in terms of microwatts it is easy to remember that this reference level is 100 db below one microwatt per square centimeter.

The need of adopting a reference intensity for sound intensity level measurements has been recognized and a tentative proposal was made in the first report of this committee. Since that time there has been considerable discussion as to whether the intensity level scale originally proposed or a pressure level scale using one bar as the reference level should be adopted. Several memoranda were written on the

points of view, which are available for any who wish to study them. After careful consideration of all the available information bearing upon this question, the committee finally voted for the recommendations given in this report. It may be noted that the adoption of these standards will make the intensity level of the 1,000-cycle reference tone the same as its loudness level. An examination of the curves in Fig. 1 shows that this is true also for pure tones covering a wide range of frequencies and intensities.

Inasmuch as the value of the intensity level of the equally loud reference tone depends upon the manner of listening to the unknown sound and also of the standard of reference, these must be specified. The manner of listening to the unknown sound may be considered as part of the characteristic of that sound. The manner of listening to the reference tone which is recommended for adoption is that given in recommendation 7.

Some members of the committee believed that there was a particular need for an additional loudness scale based upon a group judgment as to what constitutes a certain percentage reduction in the loudness of a sound. A basis for such a scale is available in the paper "Loudness, Its Definition, Measurement and Calculation," by H. Fletcher and W. A. Munson, reported at the May 1933 meeting of the Acoustical Society of America and to be published in the *Journal* of that society. The committee believed that a scale based upon this work should be proposed tentatively, in order to facilitate the interpretation of experimental work of this kind. Loudness is the quantity that probably is correlated closely with the percentage estimates of loudness reduction that a group of observers make. For example, a loudness of 8,000 corresponds to a loudness level of 70 db. A reduction of 50 per cent to a loudness of 4,000 causes a change in loudness level of 11 db.

# Power Transmission and Distribution—1932-33

**Although the expansion of electric power transmission and distribution systems has been curtailed greatly during the past year, some unusual cable installations have been made. These and other advances are outlined briefly in this report by the Institute's committee on power transmission and distribution.**

IN SPITE OF business conditions, power transmission and distribution activities have been maintained during the past year. Much of the work of the power transmission and distribution committee has been divided among and carried on by several subcommittees. Activities of these subcommittees during the past year are summarized briefly in this report.

## Steel Transmission Towers and Conductors

The work of this subcommittee has been subdivided into 3 groups covering: (1) steel towers, (2) Clearances and Electrical Characteristics, and (3) Conductors. The following items are under consideration.

1. Hinged and rigid crossarms.
2. Straight line compression formula for columns.
3. Fatigue indications at clamps.
4. Rotated towers.
5. Recommended form for service records.
6. Clearances as affected by heating of conductors.
7. Vibration of transmission conductors.

This subcommittee has been in touch with the investigation of embrittlement of hot dipped galvanized steel, which has been covered by a report to be found in the *Proceedings* of the American Society for Testing Materials, v. 32, Part II, 1932. Study is now being given to prestretching of A.C.S.R. cables. A report entitled "Modern Steel Tower Transmission Lines" was published in Electrical Engineering for April 1933, p. 243.

## Distribution

The subcommittee on distribution has been working intensively on the preparation of a coördinated group of papers for a session at the 1934 winter convention. It is proposed that this program will cover certain phases of the economics of electric power distribution based upon actual areas in the districts of several operating power companies.

## Cable Developments

The amount of new cable construction work during the past year has been rather limited. There have been no electrical failures on any of the 132-kv oil-filled cable lines operating in New York City and Chicago, Ill. The experimental 132-kv lines in Chicago and Newark, N. J., have continued without incident for another year, except for one joint failure

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Essentially full text of the annual report of the A.I.E.E. committee on power transmission and distribution for 1932–33. *Not published in pamphlet form.*

Committee on power transmission and distribution, 1932-33: P. H. Chase, *chairman;* R. N. Conwell, *vice-chairman;* T. A. Worcester, *secretary;* F. E. Andrews, G. M. Armbrust, H. W. Bibber, D. K. Blake, E. S. Bundy, A. B. Campbell, C. V. Christie, W. A. Curry, O. G. C. Dahl, A. E. Davison, R. D. Evans, F. M. Farmer, C. L. Fortescue, T. H. Haines, Edwin Hansson, C. F. Harding, K. A. Hawley, L. F. Hickernell, C. R. Higson, J. P. Jollyman, A. H. Lawton, H. L. Melvin, J. S. Parsons, F. W. Peek, Jr., L. L. Perry, D. W. Roper, H. J. Schols, H. R. Searing, A. E. Silver, D. M. Simmons, C. T. Sinclair, L. G. Smith, H. H. Spencer, Philip Sporn, W. K. Vanderpoel, and H. S. Warren.

in Chicago. Additional 35-kv oil-filled cables have been installed in Los Angeles, Calif.

The 2 outstanding high voltage submarine cable installations operating at 69-kv have continued in successful operation: one across the Delaware River, near Wilmington, Del., of the "solid" type, for $3\frac{1}{2}$ yr; the other across the Columbia River, near Portland, Ore., of the oil filled type, for about 1 year.

Two unusual submarine cable installations have been made during the year. The first is across the East River, New York City, consisting of 14 cables each, 2,330 ft long, laid 2 at a time by a novel method of installation in a narrow trench, part blasted out of solid rock. Half of the cables are of the oil filled type and half are of the "solid" type. The other installation consists of 15 cables, installed 5 at a time, from the Hudson Avenue Station, Brooklyn, N. Y., across Wallabout Bay, each 3,300 ft long without joints, the cables all being of the "solid" type. Both installations consist of 3-conductor, 500,000-cir mil paper-insulated 27.6-kv cables.

A recent installation of single-conductor 69-kv paper-insulated cable involves conductors 2,100,000 cir mils in cross section; the skin effect was reduced by making the conductor out of several sectors of stranded conductor, the sectors being lightly insulated from each other. The conductor is wrapped with copper tape to give a smooth electrical surface.

There has been considerable activity in the development of cable without lead sheath for medium and low voltage distribution; this cable may be buried directly in the ground, or pulled into ducts.

## Interconnection and Stability Factors

This subcommittee has been relatively inactive during the year because the joint interconnection subcommittee, with which it was to coöperate, was not organized, the parent technical committees finding such organization unnecessary at this time.

## Standardization

Standardization activities in the transmission and distribution field have been proceeding at a modest pace. National existing standards and national standardizing projects under way or to be initiated are, in common with other electrical standards, being brought under the jurisdiction of the new Electrical Standards Committee. This committee has been organized to coördinate all standardization of a national character in the electrical field. The following is a brief résumé of the more important recent standardization activities in connection with transmission and distribution:

1. Parts I and II of the "Code for Protection Against Lightning" which is now an American standard, have been revised by the sectional committee, these revisions are in process of approval by the American Standards Association.

2. The American Society for Testing Materials has issued "Tentative Specifications for Insulated Wire and Cable: Performance Rubber Compound."

3. Three preferred types of impulse test waves, namely, 1x5, 1x10 and $1\frac{1}{2}$x40$\mu$ sec have been recommended by the lightning and insulator subcommittee of the power transmission and distribution committee for testing insulators. One of these standard waves, $1\frac{1}{2}$x40$\mu$ sec, has been recommended by the transformer subcommittee of the A.I.E.E. electric machinery committee for testing transformers.

4. An A.S.A. sectional committee on power switchgear has been organized, and hereafter the various revisions of the A.I.E.E. standards on switching apparatus will be made through that committee. A joint N.E.L.A.-N.E.M.A.-A.E.I.C. committee on oil circuit breakers was set up to crystallize views with reference to standardization of dimensions, duty cycles, ratings, etc.

5. Specifications for weatherproof and for heat resisting wires and cables, and for impregnated paper insulation for wires and cables recently have been approved as American standards. The A.S.A. sectional committee which has these and other standards in charge, has several other wire and cable specifications in various stages of development.

6. Proposed American standard definitions of electrical terms including a comprehensive list of transmission and distribution terms have been published by the A.I.E.E. as a progress report.

## Lightning and Insulators

Considerable progress has been made during the past year in placing impulse testing of insulators on a common basis. The 3 preferred test waves recommended by lightning and insulator subcommittee last year have been accepted generally as a basis for comparative tests of impulse strength of insulators, although in some laboratories, where difficulty was encountered in producing the $\frac{1}{2}$x5-$\mu$ sec wave, a 1x5-$\mu$ sec wave has been used with fairly comparable results where flashover takes place on the tail of the wave.

In addition to agreeing upon and using defined test waves in impulse testing of insulators, an important phase of the problem is the laboratory technique in making tests. Considerable study has been given this subject, and definite progress has been made.

Field research of lightning on actual electric lines has been continued on a much restricted basis this past year. Lightning currents that have been measured in steel towers have indicated crest magnitudes, in the upper ranges, of the order of 75,000 to 190,000 amp. These figures suggest currents in the lightning stroke approaching such values.

As foreshadowed in last year's report, impulse tests on commercial transformers in the higher voltage ranges now are being made in some cases by the manufacturer as part of the routine test procedure.

During the past year the use of enclosed protective gaps has been further tried out in an effort to take lightning disturbances off transmission lines without permitting sufficient power current to flow to cause a circuit interruption. Further operating experience will be required, however, before the reliability of these devices has been proved.

From the limited experience so far available, the use of counterpoises (buried conductors) at tower bases appears to have considerable influence in reducing lightning troubles on high voltage transmission lines. Here, again, more operating experience is necessary before general conclusions can be drawn.

The continued interest of the members of the Institute in the power transmission and distribution field is a source of great gratification to the power transmission and distribution committee. The successful functioning of the committee's activities is in large part attributed to the subcommittee organization and to the initiative and interest displayed by the members.

# Maintaining Steam Turbine Performance

**Steam turbines must be judged not only upon their original characteristics, but also upon the permanence of these characteristics and the reliability of operation over a period of years. Some of the factors affecting reliability and permanence of performance of modern steam turbines are discussed in this article, and several recently developed methods of maintaining efficiencies are given.**

By
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MEMBER A.I.E.E.
Westinghouse Elec. and Mfg. Co., Philadelphia, Pa.

THE CRUCIAL test of a steam turbine is not whether it will give the guaranteed performance of economy, governing, smoothness, etc., on the acceptance test, but whether it will continue to do so year after year with very long intervals between shutdowns for inspection.

The operating records of existing steam turbines may be regarded as good, in view of the fact that the designer has always been forced to work very close to the recognized limits of the possible. Statistical data collected by the National Electric Light Association and presented in its publication No. 234, "A Report of the Prime Movers Committee," show that 324 turbines, representing a combined capacity of about 12 million kilowatts, were out of service about 4.2 per cent of the time for the years 1931 and 1932. Of this outage, about 2.5 per cent represented general annual inspections; 0.6 per cent could be attributed to blading repairs; while the remainder, about 1 per cent, was due to miscellaneous factors, of which governors, glands, and lubrication represented the largest items. The outage due to generators, condenser, and other miscellaneous causes were 1.4, 2.3, and 0.9 per cent, respectively, for the same period. This record shows that turbines and condensers were responsible for the greatest part of the outage, a result which probably corresponds to the inherent operating difficulties of these devices. Similar records, extending over nearly 20 years, show a gradual reduction of this outage, from about 7 per cent in the early years, to the present value of about 4 per cent.

The preservation of efficiency with time is one of the important problems of turbine design. It has gradually become conventional to allow about one per cent per year as unavoidable drop in efficiency during the early years. The change in efficiency is due to a variety of causes, such as coating and corrosion of blading causing rough surfaces, erosion of low pressure blading, enlargement of clearances due to wear of sealing strips, and blade distortion in the alignment of blades and nozzles due to creep and thermal deformation, and so on. The drop in efficiency generally proceeds at a slower rate as the turbine grows older.

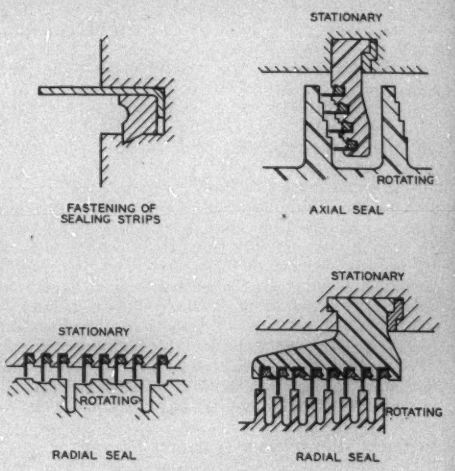


**Fig. 1. Sealing details for glands and dummies**

Two features of modern turbine design should be mentioned in this connection. These are the axial clearance type of construction, and the use of renewable sealing strips throughout the turbine. The problem of leakage of steam in the individual stages and at the end of the cylinders is one of the greatest single items of loss in modern turbines, variations in turbine design generally representing variations in leakage details. No other class of detail more forcibly affects the reliability of a turbine. The clearances which are used in modern turbines are necessarily very close, and the possibility of mechanical contact between rotating and stationary parts must be admitted. It is necessary to arrange the sealing details in such a manner, therefore, that the contact has no other consequences than a possible enlargement of the clearance. When all such adjustments have taken place throughout a turbine, it still must give its guaranteed efficiency. Experience has shown that these requirements can be met only by thin sealing strips made of special materials which will confine the effects of a contact to the strip itself. Some recent designs of these sealing details are shown in Fig. 1.

Full text of that portion of a talk presented before the National Association of Power Engineers, Buffalo, N. Y., August 30, 1933, which discusses reliability and permanence of performance of modern steam turbines. *Not published in pamphlet form.*

Of the two features of turbine design mentioned in the preceding paragraph, the axial clearance type of construction, in which the machine may be started with very large clearances, is perhaps the more important. By this feature, the critical circumstances during the starting, may be encountered with a minimum of permanent wear of sealing strips. The other feature, use of renewable sealing strips throughout the turbine, by which it is possible to reëstablish the clearances at suitable intervals, is also important from the standpoint of permanence of performance, as it permits the restoring of at least one of the causes of drop of efficiency with time.

## Erosion and Corrosion

The problem of erosion of low pressure blading is of particular importance, not only with regard to the influence upon efficiency, but also on account of the expense involved in blade repairs. This problem has been given very extensive study during recent years, with gratifying results; it was discussed in a paper, "The Moisture Problem in Steam Turbines," presented by the author at the Chicago 1933 meeting of The American Society of Mechanical Engineers. It is now generally realized that satisfactory results

**Fig. 2. An erosion tester which has enabled valuable results to be obtained in the laboratory**

for the high blade speeds can be obtained only by protecting the back of the inlet edge with shields having a hardness in excess of 500 Brinell. Stellite has been found to be one of the best materials for this purpose. By these measures, low pressure blading can now be operated at tip speeds in excess of 1,200 ft per sec with intensities of erosion no worse than formerly obtained with conventional blade steels at 900 ft/sec.

Laboratory investigations have played a very important rôle in this connection. Experiments with different materials on actual turbines gave results that were too slow and indefinite. It was not until erosion testing was developed that fully reliable results were obtained, which have since been verified in actual practice. An erosion tester is shown in Fig. 2; in Fig. 3 is a set of specimens, and in Table I the corresponding set of results. Results of the application of similar materials to an actual turbine are shown in Table II. A comparison under similar circumstances of the behavior of 12 per cent chromium stainless steel and stellite shields is shown in Fig. 4.

## Scaling of Turbines

Another problem which is gradually assuming a degree of importance, comparable to the erosion problem, is the scaling of blading on turbines operating at high pressures. This problem has come to the front in cases of difficult feed water conditions, but the indications are that it will become an unavoidable part of operation with high steam pressures.

The exact mechanism of turbine scaling is by no

**Table I—Results of Tests on Erosion Specimens of Fig. 3 With a Tip Speed of 1,000 Ft per Sec**

| Specimen | Material | Approx. Hardness Vickers Brinell | Length of Test Min | Loss of Weight % | Loss per Min % | Comparative Erosion Resistance |
|---|---|---|---|---|---|---|
| H1, H2 | 12% Cr steel with Stellite shield | 600 | 30 | 0.9 | 0.030 | 6.3 |
| I1, I3 | 12% Cr steel | 225 | 20 | 3.8 | 0.190 | 1.00 |
| J1, J2 | Tantalum | 150 | 8 | 2.7 | 0.337 | 0.56 |
| K1, K2 | 5% Ni steel | 172 | 4 | 1.7 | 0.425 | 0.45 |
| L1, L2 | Nitralloy | 200–900 | 11 | 1 | 0.091 | 2.10 |

**Table II—Erosion Record of Experimental Blade and Shield Materials**

| Material | Number of Blades | Degree of Erosion: None | Very Light | Light | Medium | Heavy | Very Heavy |
|---|---|---|---|---|---|---|---|
| Stellite | 16 | — | | | | | |
| Nitralloy | 28 | — | — | — | | | |
| Resistal No. 7 | 27 | — | — | — | — | | |
| 12% Cr St Cr Pl | 19 | — | — | — | — | | |
| 12% Cr St. | 19 | — | — | — | — | | |
| ATV No. 1 | 32 | — | — | — | — | — | — |
| 5% Ni St. | 13 | — | — | — | — | — | — |

**Fig. 3. A set of specimens after having been subjected to erosion tests. The results of tests on these specimens are given in Table I**

means fully understood, but our present conception of the process is somewhat as follows: In the evaporation process of the boiler, a certain amount of water is carried along with the steam. This water

carries with it a certain amount of salts, depending upon the concentration in the boiler. After passing through the superheater, these salts take the property of dry dust and would undoubtedly blow through the turbine if they remained in this form. At certain temperatures, some of the salt components (notably $NaSO_4$) become sticky and attach themselves to the blading. The process is cumulative and will continue, if unchecked, until the blade path is completely filled up. There are cases on record where a 15,000-kw back pressure turbine lost 1,500 kw of maximum capacity per week due to this cause. A severe case of blade deposits is illustrated in Fig. 5.

This theory of stickiness is supported by the fact that scaling always takes place over a certain definite region of temperature, which extends roughly between 400 and 600 deg F. Experiments are now being planned to check these conclusions in the laboratory.

It is evident that the most fundamental measures which can be taken to solve this problem apply to the evaporation process of the boiler. The scaling problem has been found particularly serious in installations where the condensate is not returned so that the concentration in the boilers has to be fairly high. In such cases, the feed water treatment becomes a real problem, and there are limits beyond which the purity cannot be carried. This applies to back pressure and bleeder turbines for industrial applications. It is probable that such cases are best taken care of by the use of heat exchangers (evaporators) on the exhaust side.

Up to the present time, it has not been found practicable to eliminate scaling by improvement of the steam evaporation process. It is still necessary, therefore, to find satisfactory methods for removing scales in steam turbines. The first requisite is to so treat the feed water so that the scales are soluble in water. This is obtained by ordinary water treatment methods. Non-soluble deposits on turbine blading may lead to untold difficulties. The soluble salts are removed by a washing process which is now becoming fairly well established. The temperature of the steam is lowered gradually by injecting water into the steam pipe at a considerable distance ahead of the turbine. It is evident that great care must be exercised in carrying out this process, particularly on high temperature installations. The water must be entered through specially designed nozzles in finely diffused form, and careful records must be kept of temperatures. The washing process undoubtedly represents a severe strain on the turbine, but experience has shown the method to be feasible, when conducted in the right manner.

One particularly difficult aspect of the turbine scaling problem is the corrosion of the blading and other parts, which is inevitable if a heavily scaled turbine is standing still a considerable part of the time. A very satisfactory solution of this problem is to circulate heated air through the turbine during the standstill so as to keep the deposits dry. The otherwise excellent 12 per cent chromium type of stainless steel is not impervious to the effects of ordinary salt (NaCl), which is invariably present in these deposits. This fact should be remembered in the choice of blade material for installations in this class.

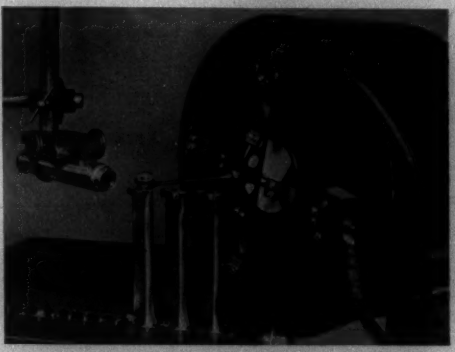

**Fig. 6. An electromagnetic vibration tester which can produce frequencies as high as 20,000 cycles per sec**

**Fig. 4 (left). Effects of erosion in an actual turbine**

**Fig. 5 (right). A severe case of blade deposits**

## Blade Failures

No item has figured more heavily in the maintenance of steam turbines than the blading. An important part of this is directly attributable to corrosion and erosion; there are many cases of mechanical failures of sound blades, however. It is becoming recognized that these failures generally can be attributed to vibrations and that they only indirectly depend upon the steady loadings due to steam or centrifugal forces. Very marked progress has been made in this respect by recent laboratory methods in detecting weaknesses of design and materials of turbine blading. An electromagnetic vibration tester in which structures can be tested to

Fig. 7 (left). A series of blade specimens tested by the apparatus shown in Fig. 6

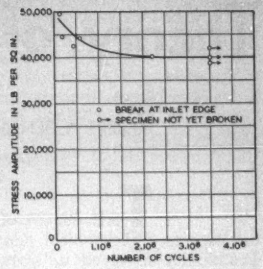


Fig. 8 (right). An example of results obtained with apparatus shown in Fig. 6

destruction under frequencies as high as 20,000 cycles per second is shown in Fig. 6. In Fig. 7 is shown a series of blade specimens tested in this manner, and in Fig. 8 is an example of the results obtained.

## Vibrations of Turbines

The vibration problem has figured prominently in turbine maintenance during the past, but the behavior in this respect of recent units has been very good. Perhaps the most important reason for this is the type of mounting of turbine cylinders which has been used on these units. A careful inspection of recent turbines will reveal that the general problem of thermal expansion is receiving increased attention. An example of such details is the mounting of the water glands at the high pressure end.

## The Starting Period

The starting period is probably the most critical part of the life of a turbine and many vibration phenomena are multiplied in intensity during this period. The adjustment of axial clearances during the starting period has been found of great value in mitigating these hardships. The use of the turning gear to prevent large distortions of the rotor (and to a lesser extent of the stator) has contributed to reduce the hazards of quick starts. Large turbines are now being equipped with recording instruments giving records of rotor eccentricity, pedestal vibration, axial expansions, and other features of interest to the operator.

## Fire Hazard

During recent years, there have been several destructive fires in turbine installations, which have focused the attention on the question of fire hazard. These fires have 2 features in common; they occurred in turbines of fairly high steam temperatures and the principal source of destruction was through burning of lubricating oil. In most instances, the principal damage occurred in the station structure, particularly the roofing, the turbine itself often miraculously escaping severe damage. In no instances were members of the operating force hurt seriously.

The close proximity of hot steam parts and combustible oil in the valve operating gear is undoubtedly one of the greatest fire hazards in the modern high temperature installations. Whether the valves are placed on top of the cylinders or in separate steam chests at the side of the cylinder does not seem to be of fundamental significance, because serious oil fires have occurred on both types of arrangements. The former arrangement may probably contribute to more extensive damage of the turbine parts.

In endeavoring to find an arrangement which will fundamentally reduce the fire hazard, a great many schemes have been proposed. Valve operation by mechanical linkages from operating pistons located at appreciable distances from the valves, is always a possibility, but the resulting complications in certain types of turbines are very objectionable.

The most attractive solution is a non-combustible lubricating medium. There are already compounds in the market for which satisfactory lubricating properties are claimed, but it is too early as yet to tell whether these claims can be substantiated. The very elaborate requirements at present placed upon lubricating oil would seem to make the finding of a synthetic non-combustible compound nothing short of a miracle. Pending the final outcome of this work, it is possible to separate the governing and lubricating systems. Many liquids may be used as governing fluid, since the requirements are so much simpler. Water may eventually become possible although the corrosion problems are somewhat difficult to anticipate. At the present time, there are a few turbines in operation in which "Aroclor" is used as a governing fluid in this manner. This compound, which is obtained by chlorination of diphenol also may be found to have satisfactory properties for use as a lubricating liquid.

In addition to these fundamental measures, a great many more obvious improvements are being made to reduce the fire hazards of turbines. The piping arrangement is given careful attention both as to arrangement and details, hot steel surfaces are carefully covered up and drip pans are introduced to direct the flow of oil from certain probable sources. In some installations, the oil tanks are placed in a fireproof housing at some distance from the turbine, and in others the tanks have been equipped with arrangements for quick drain in case of fire.

# Theory of Probability

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**Theory of probability has found many applications, some of a practical nature, others less elementary such as in the development of statistical theories of matter. There is a mathematical theory of probability, the rules of operation of which are generally acceptable and lead to results of increasing scientific and economic significance; however, the theory still is considered to rest upon an unsatisfactory philosophical basis. This article presents a brief outline of the history of probability theory and more detailed discussions of present concepts.**

THE subject of probability has various aspects, philosophical, mathematical, utilitarian. Probability involved in surmises as to future developments enters into daily conversation as friends discuss tomorrow's weather, economic forecasts, or political prophecies. Some use the word to indicate a subjective state of mind as opposed to an objective characterization of events. Qualitative grades of estimates even may be assigned in some such scale as (1) impossible, (2) highly improbable, (3) possible, (4) improbable, (5) uncertain, (6) probable, (7) highly probable, (8) almost certain, (9) certain.

There is, however, a mathematical theory of probability.[1] Save in the so-called "geometrical probability," the rules of operation are generally acceptable and lead through computation to numerical results of increasing scientific and economic significance. However, the theory still rests upon an unsatisfactory philosophical basis. First, the historical origin of the subject will be outlined briefly.

The theory of probability found its first impetus in the questionable motives of a notoriously superstitious and irrational group, namely, one of inveterate gamblers; and as a consequence, even today perhaps some secluded pupils may study playing cards and dice for the first time in a course in algebra. Although games of chance may be as old as civilization, probably the first mathematical work discussing a problem in gambling, is Pacioli's "Sūma" (1494) proposing the problem of the equitable division of states in an interrupted gambling game. Thereafter problems concerning dice or other games of chance were treated by Cardan (1539) Tartaglia (1556) Galileo (century 1600). The subject received widespread scientific recognition in view of published correspondence between Pascal and Fermat about 1654. A tract upon the topic was printed by Huygens in 1657. Early in the following century several distinguished mathematicians, raised the status of the mathematical theory of probability from that of a causal discussion of isolated problems to one of an established science. Mention may be made of Jacques Bernoulli, "Ars Conjectandi" (1711) De Moivre, "Doctrine of Chances" (1718) and Simpson, "Laws of Chance" (1740). Later famous names to be reckoned among its exponents, are Laplace, Poisson, Gauss, and recently Poincaré, Borel, Pearson, and Castelnuovo; but the writers at present in this field are legion.

As one might expect, certain early proposals have been subjected to such severe criticism during past decades that they now are largely discarded. For example, Jacques Bernoulli would regard probability as a measure of the strength of our expectation of a future event. Such a phychological definition has found recent champions among philosophers,[2] but seems to lead to the mathematically sterile conclusion that this concept is incapable of strict numerical treatment. A definition that presupposes numerical evaluation of causes[3] also is unacceptable. Any effort to study games of chance numerically is itself attacked, as by the great scientist Buffon, who (following Daniel Bernoulli) declared "A mathematician in his calculations counts money

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1. There are few texts in English devoted to the mathematical theory of probability, although the elementary part is included in most texts on college algebra and in some texts on statistics. This is in contrast to the situation in the French and German languages in which are found numerous excellent treatises. Among works of the last century are those of:

DeMorgan, "An Essay on Probabilities," 1838.
I. Todhunter, "A History of the Mathematical Theory of Probabilities," 1865.
W. A. Whitworth, "Choice and Chance," 1867 (and many later editions).
R. A. Proctor, "Chance and Luck," 1880 (chiefly moral anecdotes).

Recent works in English include:

Arne Fisher, "The Mathematical Theory of Probabilities," 1915 (chiefly statistical theory).
Marsh Hopkins, "Chance and Error," 1923 (elementary).
J. L. Coolidge, "Probability," 1925.
T. C. Fry, "Probability and Its Engineering Uses," 1928 (476 p.).
W. Burnside, "Theory of Probability," 1928.
Encyclopedia Britannica, 13th and 14th editions, have authoritative articles.

Of the numerous standard works in French and German, may be mentioned:

J. Bertrand, "Calcul des Probabilites," (2nd ed., 1907).
R. de Montessus, "Lecons Elementaires sur le Calcul des Probabilites," 1908.
Henri Poincare, "Calcul des Probabilites," 1912.
Emile Borel, "Elements de la Theorie des Probabilities," 1924.
Paul Levy, "Calcul des Probabilites," 1925.
L. G. du Pasquier, "Le Calcul des Probabilites," 1926.
A. A. Markoff (translator, H. Liebmann) "Wahrscheinlichkeitsrechnung," 1912.
E. Czuber, "Wahrscheinlichkeitsrechnung," 2 v., 3d ed., 1914.
Otto Knopf, "Wahrscheinlichkeitsrechnung," 2 v., 1923.
R. V. Mises, "Wahrscheinlichkeitsrechnung," 1931.

Some of the historical remarks in the present article are taken from D. E. Smith, "History of Mathematics," v. 1.

2. Such as Keynes, "Treatise on Probability," London 1921, and many followers.

3. Such as proposed by John Stuart Mill, "Logic," 8th ed., v. 2, p. 72, London 1872.

only as to quantity, that is to say by its numerical value, but the moral man must value it for the advantages and pleasures which it may procure," attempting thus to dispose of the annoying St. Petersburg paradox. The economic laws of diminishing returns and of marginal utility undoubtedly serve to render the gambler's inevitable ruin the more disastrous, unless the thrill of gambling and the moral experience of loss are regarded as compensating features.

By simple paraphrasing, a problem often may be restated to read as a question concerning distributional frequency, thus obviating a philosophical difficulty. The simple problem "What is the probability that if I draw a card at random from a deck of 52 playing cards, it shall be a black ace?" may be rewritten to read "What is the proportion of black aces to the total number of cards in a deck of 52 playing cards?" The subject does not degenerate thus to mere routine, however, but continues to be a test for clear thinking, some of the older problems remaining of perennial instructive interest.

Concepts and methods eminently adapted to deal with present facts come naïvely to mind; but there are difficulties beyond those that reside in the mere idea of an infinite sequence. For example, one may discuss without fear of error such an unending array as 1, 2, 3, . . . $n$, . . . (the sequence of natural numbers) or 1, $1/2$, $1/4$, . . . $1/2^n$, . . . (where each term upon division by 2 yields the next). (Here and in a later series $n$ takes on the successive values 0, 1, 2, etc.) Where a law is stated determining the method of generating successive terms, it is possible to restrict attention to the implications of the generating law. However, for probability it is preferable to deny the possibility of knowing any such law. Each event individually happens "at random," and must by hypothesis remain completely unpredictable. Unlike a purely mathematical sequence, the temporal distinction between what exists and what is yet to be appears vital for these aspects of probability.

The following parable of a form beloved by the ancient Sophists, may suggest the possibilities of paradox:[4] Some giants inhabiting an island sacrificed each stranger landing on their shore, to one or the other of 2 idols called, respectively, Idol of Truth, and Idol of Untruth. The prisoner would be asked a question. If the answer were correct, he was sacrificed to the Idol of Truth; if incorrect, to the Idol of Untruth. Such was the law of the giants, which was to be strictly observed. Now having foolishly asked a man more subtle than themselves, the question "To which idol are you about to be sacrificed?" this man replied "To the Idol of Untruth." By this answer he made it impossible for the giants to sacrifice him to either idol, for by sacrificing him to the Idol of Untruth, his reply would be rendered correct, and this would require that he be sacrificed to the other idol, and inversely.

There remain other philosophical questions. The authority for inferences as to probability does not lie in mere ignorance of the determining causes. Ludicrous consequences result if it be supposed that in the total absence of information we are justified in assigning the probability of $1/2$ to each of 2 mutually exclusive, but together exhaustive, outcomes.[5] In the ideally simple cases of drawing a card from a pack or spinning a coin, statistical observations check well with a common-sense estimates of equal likelihoods, although of course if the cards were identical in printing, or the coin polished on both sides to absolute symmetry, the outcome never could be distinguished.

Whence comes confidence in the impartiality of blind chance? Doubtless all would concur with the good Abbot Galiani, who one day observed a man shake 3 dice in a box, and throw a triple 6. The fellow succeeded a second time, and then repeated the same performance; 3, 4, 5 times, he shook the dice in the box, and each time threw a triple 6. "By the blood of Bacchus," cried the Abbot, "the dice are loaded!" They were![6] Despite the fatalism and anthropomorphic imagery of incorrigible gamblers, the tossed coin "has neither conscience nor memory."[6] The philosopher properly may inquire as to the scope of general methods. The formal mathematician who proposes exercises of his own choosing and explains their solution, may sidestep this basic question. To be on the safe side it is well to reserve the technical study of probability to classes covering many substantially uniform cases. The possibility of free experimentation is a desirable but not essential feature. We may speak of the probability of infantile mortality, the probable number of stars of a given magnitude in a portion of the heavens of given angular area, the most probable true value indicated by a large number of discordant micrometer readings, the probable life span of a white boy (treated generically) of 10 years of age in the United States, etc.; but one should not attempt to include in a mathematical study, such isolated outcomes as the probability of a war between 2 given countries during the next quarter century, the probable date of death of a given distinguished personage (save with respect to his inclusion in some stated class) or the probable accuracy of a theory. Such matters may fall within the scope of business forecasts and of private betting, but on the basis for the odds offered is at best not more than shrewd common sense.

One is tempted to inquire whether the laws of probability are necessary to thought processes, are consequences of the rules of mathematics, are traditional wisdom distilled from the experience of the race, or are experimental laws established by laboratory tests; but these debatable suggestions will not be pursued further here.

## Finite *A Priori* Probability[7]

In the theory of finite *a priori* probability, it is assumed that: An unlimited number of trials may be performed under essentially the same conditions; either 1 of only 2 distinct stated outcomes occurs in any 1 trial, these outcomes being denoted respec-

4. Due to F. Gonseth, "Les Fondements des Mathématiques," Paris 1926.

5. Despite the ironic strictures of the scientist and philosopher, d'Alembert.

6. Bertrand, "Calcul de Prob.," 2d ed. (1907) preface, p. 7.

7. Excellent formal discussions of this subject may be found in Fine, "College Algebra," 1901, and in Hall and Knight, "Higher Algebra" (many editions), and other books on college algebra, as well as in works mentioned in the first footnote.

tively as "favorable" and "unfavorable"; and the ratio of the number of favorable outcomes to the number of trials performed approaches a limiting value (called the probability) as the number of trials is increased indefinitely. A critical reader well may find this assertion to be bristling with insurmountable difficulties or at least ambiguous with unexplained terms. But it is necessary to pass without comment to even worse: If there are a certain number of equally likely ways for a given favorable outcome, then the probability of a favorable outcome is the ratio of the number of favorable ways to the total number.

Galileo considered a game of chance involving throwing 3 dice. A friend of his, long practiced in such games was astonished[8] to note that the sum 11 turned up more often than 12, and the sum 10 more often than 9. He argued that since each of these sums could occur in just 6 ways, each therefore should be equally probable. The respective sums are:

| 9 | 10 | 11 | 12 |
|---|---|---|---|
| 6 + 2 + 1 | 6 + 3 + 1 | 6 + 4 + 1 | 6 + 5 + 1 |
| 5 + 3 + 1 | 6 + 2 + 2 | 6 + 3 + 2 | 6 + 4 + 2 |
| 5 + 2 + 2 | 5 + 4 + 1 | 5 + 5 + 1 | 6 + 3 + 3 |
| 4 + 4 + 1 | 5 + 3 + 2 | 5 + 4 + 2 | 5 + 5 + 2 |
| 4 + 3 + 2 | 4 + 4 + 2 | 5 + 3 + 3 | 5 + 4 + 3 |
| 3 + 3 + 3 | 4 + 3 + 3 | 4 + 4 + 3 | 4 + 4 + 4 |

The friend observed the sum 11 turn up 1,080 times to 1,000 times for the sum 12, and hence concluded that the sum 11 is in fact more probable than the sum 12. This illustrates the application of the first assumption stated. Galileo with a more analytic mind sought an explanation in accordance with the second assumption. He showed that of the equally probable types of throws, instead of 6 there are actually 27 yielding the sum 11, and only 25 for the sum 12. He likewise showed that the chances are even of obtaining a sum in excess of 10. One quite natural argument in this connection is that since each individual die turns up a number of points not exceeding 3 as often as exceeding 3, therefore for the 3 dice together a sum not exceeding 9 would be as common as a sum exceeding 9. This argument is erroneous, however, as detailed listing of equally probable throws reveals.

The probability of an event usually is denoted by $p$, so that $0 \leq p \leq 1$, and $1-p$ (the probability of non-occurrence) usually is denoted by $q$. If an event[9] is certain to happen, its probability is 1; if it is impossible, its probability is 0.

The arithmetical theory of finite probability makes use of the notions of permutations and combinations as these are treated in algebra. A simple problem in permutations is the following: In how many ways can all the letters of the word "Mississippi" be arranged?[10] Were the 11 letters treated as being distinct, the answer would be $11! = 1 \times 2 \times \ldots \times 10 \times 11 = 3{,}991{,}680$. However, the 4 letters "i" are considered to be indistinguishable, as are the 4 letters "s," and the 2 letters "p." Thus the number of possible arrangements is only $11!/[(4!)^2 2!] = 34{,}650$.

The addition and multiplication of probabilities for so-called "compound events" follows the rules: If an event can happen in at least 2 or more ways that are mutually exclusive, the probability that it will happen in some one of these ways is the sum of the corresponding probabilities; the probability of the simultaneous occurrence of 2 or more independent events is the product of the respective probabilities. Derived rules sometimes result in considerable economy in application. For example: If the probability that an event will occur in a single trial is $p$, the probability that it will occur exactly $r$ times in the course of $n$ trials is $\binom{n}{r} p^r q^{n-r}$, while the probability that such an event will occur at least $r$ times in the course of $n$ trials is $p^n + \binom{n}{1} p^{n-1} q + \binom{n}{2} p^{n-2} q^2 + \ldots + \binom{n}{r} p^r q^{n-r}$. Here $\binom{n}{r}$ is the binomial coefficient, $n!/[r!(n-r)!]$. A powerful theorem is the following: If of $n$ different events, the probability for the simultaneous occurrence of the $i$th, $j$th, $k$th, etc., be denoted by $p_{ijk\ldots}$ (where $p_{ij} = p_{ji}$, $p_{ijk} = p_{jik} = p_{ikj} = p_{jki} = p_{kij} = p_{kji}$, and so forth) then the probability for the occurrence of at least one of these events is given by $\Sigma p_i - \Sigma p_{ij} + \Sigma p_{ijk} - \ldots$ (if each summation extends over all formally distinct terms only).

These few formulas suffice for most of what can be done by direct calculation. For more complicated problems in finite probability, indirect methods must be resorted to.[11] As illustrative of compound events may be mentioned the following: If 13 persons take their places at a round table, the odds are 5 to 1 against 2 given persons sitting together. For if 1 of the 2 be given a position at the table, of the 12 possible places for the other, exactly 2 would be favorable. If 5 cards be drawn from a full pack of 52 playing cards, the probability of their being in sequence in a single suit is less than 1 in 72,000. For the number of distinct hands of 5 cards is $\binom{52}{5} = 52!/(47!5!) = 2{,}598{,}960$, and the number of distinct 5-card sequences is only $4 \times 9 = 36$. There is more than 1 chance in 4 that a hand at bridge will hold at least 2 aces.[12] Indeed the respective chances of holding 1, 2, 3, or 4 assigned aces are $1/4$, $1/17$, $11/850$, $11/4{,}165$; but the number of ways of choosing 2 aces out of 4 is 6, of choosing 3 aces out of 4 is 4, of choosing all 4 aces is 1. If $p_i$ is the probability of holding $i$ aces, then $\Sigma p_1 = 1$, $\Sigma p_2 = 6/17$, $\Sigma p_3 = 44/850$, $\Sigma p_4 = 11/4{,}165$. Hence (with this change in notation) by the formula given previously,

8. Bertrand, preface, p. 6–7. It is interesting that Leibnitz makes the same type of error, "Opera Omnia" (Dutens) v. 6, Part 1, p. 217.

9. The converse of these statements apply under the present discussion to "finite probability," so that here every event that is neither certain nor impossible has a probability between zero and unity. Since, however, $p$ is merely the limit of a ratio, the limit may be zero (for example) without the variable ratio itself always being zero. Thus if it be assumed that in some game every natural number is equally probable, the probability of drawing a specified natural number cannot be other than zero, although it is not impossible. Similarly the probability of drawing a number, say, greater than a million, is unity, although obviously not certain. Even more strikingly in the case of continuous probability in one argument, the probability of each specified abscissa is zero. To obtain a non-zero probability, one should consider the probability that an abscissa to be chosen will lie between specified bounds on the axis.

10. Coolidge, "An Introd. to Math. Prob.," Oxford, 1925, p. 15.

11. Compare Burnside, *loc. cit.*, p. 12, and Chapter 3, "Indirect Methods."

12. Burnside, p. 16 (where the numerical computation is, however, in error).

the probability of holding *at least* 1 ace is $1 - \frac{6}{17} + \frac{44}{850} - \frac{11}{4,165} = 0.6962$, approximately. From this must be deducted the probability of holding exactly 1 ace which is 0.4388 approximately, as is seen for example by dividing $4\binom{48}{12}$, the number of distinct ways of holding 1 ace and 12 cards each other than an ace, by $\binom{52}{13}$ the total number of distinct ways of drawing 13 cards from a full deck. The probability of holding at least 2 aces is then 0.257 + or better than 1 in 4.

As shown by de Montmort (in 1713): If from an urn containing $n$ balls numbered 1, 2, . . . $n$, balls be withdrawn one after another at random, the probability that no ball will appear in the turn corresponding to its number is $1 - \frac{1}{1} + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^n \frac{1}{n!}$. (By definition $0! = 1$, agreeing with the recursion relation $(n + 1)! = (n + 1)\,(n!)$ for $n = 0$.) This, however, is the series obtained by breaking off the rapidly convergent expansion for $1/e$, where $e$ is the base of natural logarithms, and is equal to 2.718. . . The situation may be stated more picturesquely by asserting that: If a careless clerk has a large number of addressed letters and corresponding addressed envelopes, and places the letters in the envelopes at random, the odds are better than 5 to 3 that at least one letter is placed in its correct envelope.

### The Case of Oft-Repeated Trials

Let $n$ be the variable denoting the number of trials performed under essentially identical conditions, the probability of whose favorable outcome is $p$. Let $f_n$ be the number of times in $n$ trials that the outcome is found to be favorable. The most probable number of favorable outcomes in $n$ trials is naturally the integer nearest to $np$. A properly balanced die has the probability $^1/_6$ of showing a given number of points from 1 to 6, in one throw. It would not be surprising to have sometimes exactly 4 out of a given series of 24 throws show a 6-spot; but no one would expect that out of 24,000 throws, exactly 4,000 should show each a 6-spot. The law (based on work of James Stirling) states: The probability that the ratio of favorable outcomes shall be the most probable ($np$) in a sequence of trials under essentially identical conditions is inversely proportional to the square root of the number of trials.

Therefore, *discrepancies* must be expected between the best predicted value ($np$) of the number of successful outcomes and the observed number $f_n$. Furthermore, these discrepancies, although clustering near zero in the main, show no tendency to remain bounded. Indeed, suppose a probability $p_0$ be specified in advance; what are the bounds $(np - l,\ np + l)$ within which there is the probability $p_0$ that every $f_n$ will lie? In answer to this: The length of interval, $2l$, within which there is an assigned probability of the (absolute) discrepancy lying, increases in proportion to the square root of the number $n$ of trials. It must not be inferred, however, that this asserted probable increase in the bounds for (absolute) discrepancy is inconsistent with the assumed approach of the ratio $f_n/n$ to $p$ as a limit. In fact when the concept of *relative discrepancy*, namely $(np - f_n)/n$ or the numerical difference between $p$ and $f_n/n$, is introduced, the length of interval within which there is an assigned probability of the relative discrepancy lying decreases in proportion to the inverse square root of the number $n$ of trials.

If, for instance, the value of the probability $p$ of the favorable outcome of some trial be based upon results of experiment, and an estimated figure be obtained upon the evidence of some given number of trials, then to secure one more significant figure in the estimated value of $p$, means performing approximately 100 times as many trials.

### Probability of Causes; Probability *A Posteriori*

The physicist, biologist, economist, cannot be sure of causes and hence of derived probabilities until after enough events have occurred to justify inductive inferences. To spin a coin twice, and for it to show heads both times, need not cause astonishment. Buffon threw a coin 4,040 times and observed that 2,048 times it showed heads. Does this imply a slight unbalance in the coin or partiality in the method of throwing, or does this result seem entirely consistent with an impartial chance? On this evidence, what is the probability that the coin was not "good?" To handle questions of this sort one is led, however reluctantly, to make use of Bayes' principle first propounded in 1763, and of developments therefrom.

Assume, for example, that a given observed condition arises, and that its occurrence may be ascribed as subject to any one of a given finite set of mutally exclusive causes, say $C_1, C_2, \ldots, C_m$, one and only one of which therefore must have been operative. Let $\pi_1, \pi_2, \ldots \pi_m$ be their respective *a priori* probabilities, so that $\pi_k$ is the limit approached by the ratio of the number of times that cause $C_k$ operated, to the total number of trials. It is not required that cause $C_k$ always should result in the occurrence of the observed phenomenon, but that rather when $C_k$ is operative, the probability of the event is to be $p_k$. One classical formulation is in terms of black and white balls in urns. The separate urns serve in one illustration to represent equally likely causes. An urn is selected at random (say blindfolded). The act of drawing a ball brings into play the $p_k$, which is the ratio of the number of balls of the desired color in the given urn to the total number of balls in this specified urn. Bayes' principle states that: When a favorable outcome is observed, the probability that the operative cause was $C_k$ is

$$\frac{\pi_k p_k}{\pi_1 p_1 + \pi_2 p_2 + \ldots + \pi_m p_m}$$

Consider the following problem:[13] An urn contains $n$ balls each of which is either white or black

13. Compare Burnside, *loc. cit.*, p. 56. The discussion here given is adapted from that text.

and each of which is equally likely to be drawn. A ball is drawn at random and is found to be white. It is returned; and a second ball is drawn. What is the probability that it is white? Consider the possibility that exactly $k$ of the balls are white. Let the *a priori* probability that this is the case be denoted by $\pi_k$. This would mean that for fixed $n$, out of a large number of repeated trials of this sort, the agent who in each case prepared the urn, placed exactly $k$ white balls in the urn approximately $\pi_k$ (e. g., $^2/_3$) of the times. Then $k/n$ is the $p_k$ of the formula, and $k\pi_k / \sum_{k=1}^{n} k\pi_k$ is the *a posteriori* probability that $r$ of the balls are white. Multiplying this by $k/n$ gives $(\Sigma k^2 \pi_k)/(n \sum_{k=1}^{n} k\pi_k)$ as the desired probability that the second ball be white. However, the numerical answer will depend upon the assumption made as to the various probabilities $\pi_k$. If it be assumed that the agent was equally likely to put in 1, or 2, or 3, . . . up to $n$ white balls, then $\pi_k = 1/n$, and the numerical answer is $(2n + 1)/3n$. If, however, each ball is equally likely as not to be white, the probability of exactly $k$ white balls is $\binom{n}{k}2^n$, as obtained from the binomial expansion. The numerical answer in this case is $(n + 1)/2n$. Difficulties of this sort have helped to discredit the principle.

## Probability in a Continuous Distribution

When measurements are made with respect to some specified characteristic of a large class of closely related but not wholly indistinguishable specimens, 2 sorts of cases arise according as the measurement is by nature discrete or measures a variable that varies over a continuous range. For instance, a count of tail feathers in pigeons of a given breed may show variations, but necessarily by integral steps, while heights, weights, etc., for classes of the population illustrate the latter case. The continuous variable may be denoted as being measured by $x$. For a given choice of $x$ and a given choice of increment $\Delta x$, the number of individuals whose measurements fall in the interval $x$ to $x + \Delta x$ will depend upon both $x$ and $\Delta x$. When the measurements have been carried out to a fine degree of precision and their number $N$ has been large, the number in this interval will be represented adequately, for small $\Delta x$, by a product of the form $f(x)\,\Delta x$, being proportional to $\Delta x$ and to a so-called *frequency function* $f(x)$. Here $f(x)$ is not negative. Furthermore (for any finite number of measurements) the range of variation shown with respect to $x$ is finite. In usual cases, if this range be subdivided into equal intervals each of length $\Delta x$, then each such interval multiplied by a suitable value for $f(x)$ in the interval, say, $f(x_i)$ provides one term $f(x_i)\,\Delta x$ in the sum $\sum_{i=1}^{n} f(x_i)\,\Delta x$ which latter yields approximately $N$, the total number of individuals measured. For suitably chosen uniform scales (varying with $N$) a sequence of broken line graphs may be obtained which in standard cases fit more and more closely to a limiting curve. The theory of such distributions in general and of their most significant characterizing parameters falls under the larger subject of "statistics."

As has been seen, $y = \binom{n}{x}p^x q^{n-x}$ represents the probability that an event (whose probability in a single trial is $p$) will occur exactly $x$ times in $n$ trials. For a fixed $n$, and for $x$ ranging over integers from 0 to $n$, $y$ represents a *point-binomial* graphically exhibiting the relative probabilities of various numbers of successful outcomes for a run of $n$ trials. If scales for abscissas and ordinates be varied with $n$ (involving in fact $\sqrt{n}$) the point-binomial may be made to approach a determinate limiting curve of finite maximum ordinate and area, but unlimited range. This limiting curve is the justly celebrated *normal curve* (of error) whose equation may be written variously, for example (for suitably named units) as

$$y = \frac{1}{\sqrt{2\pi}} \frac{A}{\sigma} e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$$

where $A$ is the total area under the curve, and $\sigma$ is the *standard deviation*. The curve is symmetrical about the maximum ordinate

$$y_0 \frac{1}{\sqrt{2\pi}} \cdot \frac{A}{\sigma} = 0.39894\, A/\sigma$$

It falls away on either side, reaching an inflexional point at $x = \pm\sigma$, with corresponding inflexional ordinates of $0.341345\ A/\sigma$, and thence tapers off smoothly toward the right and left, the ordinates becoming and remaining as small as desired.

In the absence of reason to the contrary, an assumed normal law for an unknown distribution has been found often to yield useful results. The entire theory of dispersion of artillery fire has been developed upon this assumption. Less elaborate but no less satisfactory have been the applications to astronomy and the general theory of precision measurements. So-called "proofs" of the law involve further simple assumptions.

If $P$ denotes the *probable error* here used[14] to represent that discrepancy from the most probable value, which in the long run is as likely to be exceeded as not, then to the nearest half per cent one has the following table of normal probability in multiples of $P$. The diagram has been called the "dispersion ladder."

| $-\infty \leftarrow -4P$ | $-3P$ | $-2P$ | $-P$ | 0 | $P$ | $2P$ | $3P$ | $4P \rightarrow +\infty$ |
|---|---|---|---|---|---|---|---|---|
| 1/2% | 1 1/2% | 7% | 16% | 25% | 25% | 16% | 7% | 1 1/2% | 1/2% |

The diagram indicates the percentage of cases falling between successive multiples of $P$, where the origin 0 is taken at the abscissa of maximum probability. For instance if $^1/_2$ of all the cases are known to fall symmetrically in a width from $-P$ to $+P$, then a width 3 times as great (from $-3P$ to $+3P$) will hold all but about 4 per cent of the cases.

## Further Developments

The computation of probabilities in drawing cards from a pack or in throwing dice would not be

14. Confusion arises from various uses for the same technical terms in the theory of probability. "Probable error," "mean error," and "absolute error" suffer from this systematic ambiguity.

worthy of serious study, were it not that these trivial problems present in simple outline the difficulties encountered in many otherwise unrelated fields.[15] The problems of congestion at a telephone switchboard, the preparation of vital statistics, the large scale production of suitable assorted shoe sizes, the efficient inspection by sampling of quantity production of minor hardware, the Mendelian theory of inherited unit characters, the accuracy of geodetic surveys, computation of the mean motion of the stars in the Galaxy, the economic margin in group insurance—these are isolated but fairly obvious applications of probability theory which establish the significance of the subject on a level above that of jig-saw puzzles.

Less elementary are the applications to principles of temperature, radiation, Brownian movement, and quantum theory based upon statistical theories of motion of molecules, ions, or what-not, in "random" distribution. Clausius, Maxwell, Gibbs, Kelvin, Boltzman, Planck, and others have established the second law of thermodynamics upon a molecular theory of gases. The entropy[16] becomes proportional to the logarithmic probability of a distribution in phase-space, deviating from the most probable one of homogeneity, and measures in some sense the element of practical irreversibility. Any one familiar with these refinements of modern dynamics[17] and recent developments in quantum mechanics will recognize the substratum of probability theory upon which the statistical developments are based.

15. Consult T. C. Fry, mentioned in footnote 1.

16. For a recent critical estimate rejecting, however, details of this classical view see R. H. Fowler, "Statistical Mechanics," Cambridge, 1929, particularly p. 137–43.

17. Consult J. H. Jeans, "Dynamical Theory of Gases," Cambridge, 4th ed., 1925. Note comparison to pack of cards, p. 182. Also L. B. Loeb, "Kinetic Theory of Gases," New York, 1927.

# Developments in Protective Devices—1932-33

**Activities of the Institute's committee on protective devices during the past year are outlined briefly in this report. New developments in circuit breakers and relays are reported.**

THE COMMITTEE on protective devices has been active during the year in connection with the sponsoring of papers for the various conventions, the preparation of reports, the revision of present standards and the preparation of new standards. The organization of the committee consisted of 4 subcommittees, as follows: fault current limiting devices; lightning arresters; oil circuit breakers, switches, and fuses; and relays.

## Fault Current Limiting Devices

There has been comparatively little activity in fault current limiting devices recently, though it is the intention of the committee to carry on the investigation of the Peterson coil or arc suppressor.

## Lightning Arresters

The lightning arrester subcommittee, working jointly with the N.E.L.A. subject committee on lightning arresters, has prepared a report covering "Present Practice in Installation and Performance of High Voltage Lightning Arresters"; this was presented at the 1933 summer convention (see ELECTRICAL ENGINEERING, v. 52, June 1933, p. 394–400).

## Oil Circuit Breakers, Switches, and Fuses

The subcommittee on oil circuit breakers, switches, and fuses has been active in the preparation of a standard for fuses above 600 volts. There has been some difference of opinion regarding the basis of current rating. A small but active minority of the engineers in the United States have advocated rating fuses on an intermediate value that is neither the carrying capacity nor the blow-out current. This subcommittee has investigated this question and finds an almost unanimous agreement throughout the United States and Canada in favor of rating fuses on the basis of carrying capacity; the proposed standard provides for rating fuses on this basis. This standard has been recommended for publication in report form.

The subcommittee also has in preparation a standard for disconnecting, horn-gap, and knife switches, but this is being held back pending completion of the work on coördination of insulation.

## Relays

The relay subcommittee has reviewed the report on A.I.E.E. Standards No. 23 for relays which has been published in report form, and, after making certain additions and revisions, has recommended the adoption of this report as a permanent A.I.E.E. Standard. The relay subcommittee also has studied

Excerpts from the annual report of the A.I.E.E. committee on protective devices for 1932–33. *Not published in pamphlet form.*

**Committee on protective devices, 1932–33:** R. T. Henry, *chairman;* H. P. Sleeper, *vice-chairman;* L. E. Frost, *secretary;* Raymond Bailey, R. C. Bergvall, H. W. Collins, A. W. Copley, W. S. Edsall, J. H. Foote, S. L. Goldsborough, S. M. Hamill, Jr., T. G. LeClair, J. P. McKearin, H. A. McLaughlin, D. M. Petter, H. J. Scholz, H. K. Sels, L. G. Smith, E. R. Stauffacher, H. R. Summerhayes, B. F. Thomas, Jr., O. C. Traver, E. M. Wood, and H. B. Wood.

modern methods of protecting apparatus and has prepared a report giving the consensus of opinion of a large number of engineers. This material was presented at the 1933 winter convention (see "Recommended Practices for the Protection of Electrical Apparatus" ELECTRICAL ENGINEERING, v. 51, December 1932, p. 829–34).

The principal developments in protective devices during the year have been on circuit breakers and relays. Further developments and improvements have been made on circuit breakers, principally along the line of higher operating speed and improved arc extinguishing characteristics. Further developments and improvements have been made on relays, particularly along the lines of higher speed and improved selectivity. Among recent developments in relay schemes are schemes using positive or negative phase sequence voltages or currents, and improved pilot relay schemes using communication circuits and carrier current circuits.

Among the principal activities for the coming year is the revision of the duty cycle for oil circuit breaker interrupting rating. The joint N.E.L.A.-A.E.I.C.-N.E.M.A. committee on oil circuit breakers presented a report at a meeting in Detroit, Mich., on April 5, 1933, which probably will be the basis of a revision of the duty cycle and of the derating factors for various operating duties.

# Power Limits of 220-Kv Transmission Lines

**For the transmission of power from Boulder Dam to the system of the Southern California Edison Company, numerous studies were made to determine the power limits of various designs of transmission lines, and the maximum generating capacity that could be installed safely at Boulder Dam without incurring instability. These studies were made for various design features and types of load and faults. The results of the studies are presented in this paper, a description of the method used being included in the appendixes.**

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THERE will be available to the Southern California Edison Company, Ltd., at the Boulder Dam project a maximum of 860,000,000 kwhr per year, which corresponds to 98,000 kw at 100 per cent load factor. This block of power must be transmitted to the load center of the company some 280 miles away. Study indicated that the best voltage for economical transmission lies between 154 and 220 kv. In considering the amount of energy available, the distance of transmission, and investment costs, it at once becomes apparent that the generating plant at Boulder Dam must be a large capacity peak plant in order to be justified at all.

A tentative installation of 250,000 kw was assumed at first. The design of the installation was varied and the safe operating limits of the transmission system were determined for various designs. The safe operating limit was considered to be that value of transmitted power for which the system will remain stable through a 2-conductor-to-ground fault cleared in 0.2 sec. A number of solutions were made involving various designs and sizes of installation and various classes and sizes of system load. As a result it was possible to estimate the size of an installation of given design which would be equal to the safe operating limit of the transmission system.

The results of the studies are expressed in Figs. 1 and 2. They can be summarized as follows:

1. For direct transmission of Boulder Dam power to the system, 2 220-kv lines tied directly into the low reactance 220-kv backbone of the system will give the highest transient stability limit.

2. The safe operating limit for a 250,000-kw generator installation would vary from 163,000 kw to 270,000 kw, depending upon how many stabilizing factors are included in the design. These stabilizing factors, in the order of their relative economy, are as follows: (*a*) impedors in the neutrals of the transformer banks at Boulder Dam; (*b*) low generator transient reactance; (*c*) a second intermediate switching station; and (*d*) increased generator inertia.

3. When the size of the generator installation is made to be equal to the safe operating limit of the transmission system it is found to range from 80,000 kw to 296,000 kw depending upon the design as explained above.

It can be seen from this summary that a relatively small additional investment goes a long way toward making the plant stable during 2-conductor-to-ground faults. Also, the method of increasing stability of operation during faults by inserting a resistance in series with generator armature windings was found to give a tremendous gain in power limits not only during 2-conductor-to-ground faults but

Full text of a paper recommended for publication by the A.I.E.E. committee on power transmission and distribution, and scheduled for discussion at the A.I.E.E. winter convention, January 23–26, 1934. Manuscript submitted April 17, 1933; released for publication August 28, 1933. *Not published in pamphlet form.*

even during 3-phase short circuits. It was felt that although there are no installations at the present time incorporating this feature, the cheapness of the device and its large stabilizing effect made it worth considering.

### Definition of Power Limit

The term "power limit" as used in this paper refers to what is considered to be the safe operating limit, allowing for the effect of such short circuits as ordinarily occur on the system. The calculations were made by the "transient stability" method, which considers the relative motion of the machinery on the system during and after a fault. This method is described in detail in the appendixes.

The criterion adopted for the safe operating limit was that the system must remain stable when subjected to a 2-phase-to-ground fault which is cleared in 0.2 sec. This speed of clearing could just about be attained at the present time with the most modern circuit breakers and the application of the carrier pilot system of relay protection; hence it is safe to assume it will be obtainable at the time the Boulder Dam lines are built. The highest possible speed of clearing is desirable not only because it has a very powerful effect in increasing stability but also because it prevents conductor burn-downs and reduces the tendency for faults which start as single-phase to develop into 2-phase and 3-phase trouble.

The use of a 2-phase-to-ground fault as the criterion for stability is reasonably conservative, because judging by experience on the Southern California Edison Company's Big Creek system, the majority of the faults will be single-phase-to-ground, and the limit for this type of fault is much higher than for the 2-phase-to-ground fault. However, a fairly large proportion of the latter may be expected on the Boulder Dam lines, especially in view of the lightning hazard, and therefore in order to maintain a good standard of service the transmitted power should be limited to such a value that these 2-phase-to-ground faults will not result in instability. As for the possibility of 3-phase faults, Big Creek experience shows 5 per cent of the total to fall in this class; but most of these faults only became 3-phase after the expiration of considerable time. Since the stability limit for 3-phase faults is far below that for 2-phase-to-ground faults, and since the probabilities are that 3-phase faults will occur very infrequently, it would hardly be justifiable from an economic standpoint to reduce the power limit to the 3-phase fault value.

### Analysis of Power Limit Curves

The results of the power limit study are presented in the form of curves whose abscissas are the times of clearing the trouble, and whose ordinates are the maximum power that can be carried prior to the fault without instability resulting therefrom, assuming the trouble to be cleared in the time indicated by the abscissa. This mode of presenting the results was adopted because it arises naturally from the method of calculation, and also because it shows very strikingly the great advantage to be gained from rapid clearing of trouble.

In Fig. 1 are presented power limit curves for 2-phase-to-ground faults at or near the Boulder Dam end of the line, with an installation of 250,000 kw at Boulder Dam. Curve *A* of Fig. 1 is based upon what might be termed the "simplest possible layout,"—that is, generators of normal characteristics, no neutral impedors at Boulder Dam, and only one intermediate switching station. It is seen that for 0.2-sec clearing the limit of generation is 163,000 kw. Adding neutral impedors at Boulder Dam increases the limit to 212,000 kw as shown in curve *B*. Decreasing the generator transient reactance from 30 per cent to 21 per cent raises the limit to 233,000 kw (curve *C*) and adding a second intermediate switching station carries it to 255,000 kw (curve *D*). Increasing the generator inertia 50 per cent above the allowable minimum would add another 15,000 kw in generated power as shown by curve *E*.

In Fig. 2 is shown a chart for 0.2-sec speed of switching of a 2-conductor-to-ground fault. The abscissas are thousands of kilowatts installed at Boulder Dam and the ordinates are transient power limits for a 2-conductor-to-ground fault near Boulder Dam cleared in 0.2 sec. The curves represent various design factors added to those represented by the curve immediately below. The diagonal line through the origin is the line representing an installed capacity equal to the transient stability limit, and its intersections with the curves of design factors give values of installed capacity as used in Table II.

Table I is based upon curves shown in Fig. 1 for a

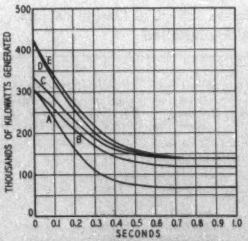


**Fig. 1 (left). Power limits of a 250,000-kw installation versus time of clearing of a 2-conductor-to-ground fault at Boulder Dam power house**

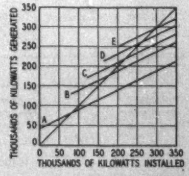


**Fig. 2 (right). Power limits of 0.2-sec clearing of 2-conductor-to-ground fault at Boulder Dam power house versus installed generating capacity**

Curves *A* based upon "simplest possible layout"
Curves *B* based upon equipment represented in curves *A* plus the addition of neutral impedors
Curves *C* based upon curves *B* plus a decrease in generator transient reactance
Curves *D* based upon curves *C* plus a second intermediate switching station
Curves *E* based upon curves *D* plus an increase in generator inertia

250,000-kw generating installation at Boulder Dam. In the first column the design factors producing improvement in power limit are tabulated in order of their cost. Each factor tabulated in column 1 is added to those shown above it, that is, the items are cumulative. The second switching station, for example, is added to a plant already equipped with transformer neutral impedors and low generator reactance. The safe generation limit is then read from curves of Fig. 1 for 0.2-sec clearing and tabulated in column 2. The power received at the load end then is computed from column 2 and line efficiency curves, the results being tabulated in column 3. The gain in power received at the load due to the introduction of the design factor is found easily by subtracting from the value in column 3 the one just above it. This gain is shown in column 4. Column 5 shows the cost of the design factor in per cent of the total cost of the project.

The outstanding items from Table I are the small percentage of the total spent for all the improvements and the large gain in power limit. The expenditure of about 3 per cent additional will increase the safe operating limit from 163,000 kw generated, or 65 per cent of installed generator capacity, to 270,000 kw, or 108 per cent of installed generator capacity.

All the above computations were carried out with an assumed installation of 250,000-kw generator capacity. The use of Fig. 2 permits the reverse process by varying the size and design of the generating station to fit the safe operating limit of the transmission system. For example: 80,000 kw is the size of generator installation of normal design which will not exceed the transient stability limit when fully loaded; and 296,000 kw is the size

**Table I—220-kv Transmission, Effect of Design Factors, 250,000 kw Installed**

| 1<br>Factor Producing Improvement | 2<br>Total Generated Kw* | 3<br>Total Received Kw* | 4<br>Gain in Received Kw | 5<br>Cost of the Factor in % of Total |
|---|---|---|---|---|
| None | 163,000 | 148,000 | | |
| Neutral impedors at Boulder Dam | 212,000 | 181,000 | 33,000 | 0.16 |
| Lower generator reactance | 233,000 | 197,000 | 16,000 | 0.38 |
| Second switching station | 255,000 | 214,000 | 17,000 | 1.00 |
| Higher generator inertia | 270,000 | 225,000 | 11,000 | 1.12 |
| Peterson coils | | | Loss | |

* Including effect of all factors preceding in the list, in addition to the one under consideration.

**Table II—220-kv Transmission, Effect of Design Factors, Variable Installation**

| 1<br>Factor Producing Improvement | 2<br>Installed and Generated Kw | 3<br>Total Received Kw | 4<br>Total Cost % |
|---|---|---|---|
| None | 80,000 | 75,000 | 100 |
| Neutral impedors at Boulder Dam | 176,000 | 160,000 | 114 |
| Lower generator reactance | 216,000 | 185,000 | 122 |
| Second switching station | 260,000 | 218,000 | 131 |
| Higher generator inertia | 296,000 | 245,000 | 137 |

installation that includes all the stabilizing factors under discussion and will not exceed the transient stability limit when fully loaded. Table II is based upon Fig. 2 and is computed in a manner similar to that for Table I. Column 1 is identical with column 1 in Table I. Column 2 shows the size of installation which will permit full load to be carried on generators within safe operating limits and is read from Fig. 2 at the intersections of design curves with the diagonal through the origin. Column 3 is computed by applying line efficiency correction to column 2. Column 4 shows the total cost of the project in per cent of the cost of the basic installation.

## Analysis of Design Factors

The "base design" assumed at the beginning of the study comprises generators of 30 per cent transient reactance and normal inertia, equipped with high speed excitation, step-up transformers of from 10 per cent to 12 per cent reactance with neutrals of the high voltage windings solidly grounded, and 2 220-kv transmission lines similar to the Southern California Edison Company's Vincent line, with one sectionalizing and paralleling switching station between Boulder Dam power house and Etiwanda 220-kv receiving substation. Six other design factors are discussed in the following paragraphs.

## Transformer Neutral Impedors

An impedance connected from the neutral of the high voltage Y connection to ground is the most effective means of increasing the safe operating limit during faults to ground near the generating station. It has little if any effect when the fault is at the load end and no effect during faults that do not involve ground returns. For the accepted criterion of stability, however, the transformer neutral impedors show the maximum improvement as compared to other design factors. Impedors of 3 times the transformer reactance or 30 per cent to 35 per cent were at first considered, but were found to limit ground currents to such a low value that it was doubtful whether even the carrier current protection would be capable of clearing the line. A lower value of impedance therefore is recommended, as it allows enough ground current to operate relays on the faulted line and still is quite effective in increasing the safe operating limit, as can be seen from a comparison of curves $A$ and $B$, in Fig. 1. The value recommended is from 10 per cent to 12 per cent, or equal to the reactance of the transformers.

## Lower Generator Reactance

It can be stated that the amount of reactance in the circuit has roughly the same effect on transient stability limits as it has on steady state stability limits; that is, the maximum power that can be transmitted through the circuit is inversely proportional to reactance. In a circuit of 95 per cent reactance a 9 per cent reduction will mean approximately a 10 per cent increase in power limit. The desirability of lower reactance for increasing power

limits is therefore quite apparent. The limitations of machine design do not permit, however, going below 20 per cent transient reactance of generators and 8 per cent reactance of transformers. The reduction assumed in this study is from 30 per cent generator transient reactance to 21 per cent generator transient reactance.

### A Second Switching Station

This proposal is to add a sectionalizing and paralleling switching station to the base design of lines, thus splitting the lines between Boulder Dam power house and Etiwanda 220-kv substation into 3 nearly equal sections. Such an arrangement is more desirable from the point of view of maintenance than it is from the point of view of stability. However, with fast switching the number of switching stations becomes a stabilizing factor. While at 0.7-sec clearing of faults the number of switching stations is immaterial, nearly 20,000 kw are gained by a second switching station at 0.2-sec clearing.

### High Generator Inertia

Variations in generator inertia affect only the time scale of curves of power versus speed of clearing the fault. The time varies as the square root of the relative inertia of the generating station and the system. For example, if the system inertia constant is 12.00 and the generating station inertia constant is 4.00, their relative inertia is $\frac{4 \times 12}{4 + 12} = 3$. Increasing the generator inertia 50 per cent makes the generator constant 6.00 and the relative inertia $\frac{6 \times 12}{6 + 12} = 4$. Hence, if 200,000 kw can be carried through a fault cleared in 0.2 sec with generators of normal inertia, then with generators whose inertia is 150 per cent of normal the same amount of power can be carried through the same fault cleared in $\sqrt{\frac{4}{3}} \times 0.2 = 0.23$ sec. As the inertia of the generators is increased its beneficial effect decreases rapidly and further increase soon becomes uneconomical. It is not recommended that the inertia be made greater than 150 per cent of normal for the Boulder Dam installation, as the next 50 per cent of additional inertia will only add some 5,000 kw to the safe operating limit of the transmission system.

### Peterson Coils

The use of Peterson coils at both ends of the Boulder Dam transmission system would be possible if the lines were isolated from the rest of the Southern California Edison system by the use of insulating transformers of 1:1 or perhaps some other ratio. With this arrangement, single-phase-to-ground faults would not exist, and 2-phase-to-ground faults would become 2-phase only. Hence in this case 2-phase faults should be considered the criterion of stability. The solid curve of Fig. 3 shows the power limits

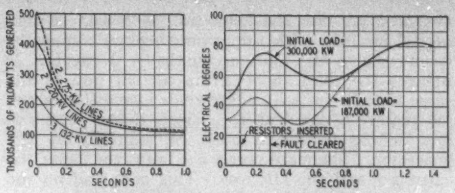


**Fig. 3 (left). Power limits of a 250,000-kw installation with Peterson coils included in the design of the transmission system versus time of clearing of a 2-conductor-to-ground fault at Boulder Dam power house**

**Fig. 4 (right). Electrical degrees versus time curves of a 250,000-kw generating plant during a 2-conductor-to-ground fault with stabilizing resistors inserted in the circuit**

under this condition, assuming 220-kv operation. This curve shows a limit of 206,000 kw for a 0.2-sec clearing. This limit was compared to the one obtained with ordinary impedors at Boulder Dam power house and no insulating transformers at the load end, other items being identical. The latter was found to be 232,000 kw for 0.2-sec clearing. Thus the limit with the Peterson coils is seen to be 26,000 kw less than that obtainable with the neutral impedors under comparable conditions, and the Peterson coil arrangement would cost vastly more than the neutral impedors. The dotted curve and the dash curve of Fig. 3 are power limits of 3 132-kv and 2 275 kv lines with Peterson coils in the neutrals of terminal transformers. The reason the limit with the Peterson coils is so low is because of the effect of the added reactance of the insulating transformers.

It is true that the Peterson coils would eliminate all single-phase-to-ground faults. However, calculations show that with neutral impedors at Boulder Dam there is no need to worry about stability on this type of fault for any installation under 275,000 kw. Hence it is safe to conclude that the possibility of using Peterson coils, at least in connection with 220-kv transmission, definitely should be rejected.

### Stabilizing Resistance

The use of a series resistance introduced in the generator armature circuit during a fault on the transmission system has been fully described.[3] When a fault occurs on the transmission system between a hydroelectric power house and the load, the fault acts as a reactor of comparatively low reactance in parallel with the load. As a result the total current of the generator increases considerably but shifts in phase so that its real (or power) component is quite small. The power house becomes overloaded in amperes and loses kilowatts. With the kilowatt output reduced and the input at its original value, the generators overspeed and fall out of

3. For all numbered references see list at end of paper.

synchronism with the load end. A resistance connected in series with the armature circuit upon the occurrence of the fault accomplishes 2 effects. It reduces the total short-circuit current and at the same time it shifts its phase angle increasing its real (or power) component to a large value. The $I^2R$ loss in the resistance must be supplied by the generator. The more severe the fault, the larger the current $I$ and the $I^2R$ loss. This new load on the generator reduces the net accelerating torque on the generator and during severe faults even acts as overload to slow it down. A 62,500-kw 13.8-kv generator would require resistors of 0.92 ohm per phase. A total of 12 resistors and 4 circuit breakers of 150,000-kva interrupting capacity would be required for the Boulder Dam power house. The comparison shown in Table III can be made to the base design installation of Table I.

**Table III—Effect of Series Resistance**

| | Factor Producing Improvement | |
|---|---|---|
| | None | Series Resistance |
| Total generated kilowatts | 163,000 | 300,000+ |
| Total received kilowatts | 148,000 | 250,000+ |
| Total cost of project | 100% | 101% |

It can be seen that a 20 per cent overload can be carried through a 2-conductor-to-ground fault when series resistors are added to the generator equipment. In Fig. 4 are shown 2 swing curves illustrating the action of series resistors. A 280,000-kva hydroelectric station connected through 2 220-kv transmission lines to a 600,000-kw load with load end generators was studied. A 2-conductor-to-ground fault was applied near the power house. In 0.1 sec after the fault was applied a 30 per cent resistance was inserted in series with the generator armatures. The fault itself was cleared in 0.3 sec. The solid curve shows the resulting swing when the initial load was 300,000 kw, and the dotted line shows the swing when the initial load was 187,000 kw. It can be seen that in both cases the system is stable with plenty to spare.

The use of series resistance alone would increase the transient stability of the Boulder Dam transmission system considerably more than any or all other design factors. This statement, however, is based on theory only. There are no installations at the present time equipped with stabilizing resistors. The tremendous gain in stability shown by calculations makes it desirable to try it out on an actual system or a miniature set-up to obtain confirmation of the theory.

## Appendix I—Description of Method

In a preliminary study of the problem of transient system stability, high speed records of 7 short circuits on the 220-kv system of the Southern California Edison Company were analyzed. Mathematical solutions necessarily require a large number of assumptions as to transient reactance, character of load, etc. Step by step solutions based upon assumptions which were to be used in further studies very closely approximated the high speed records obtained during the disturbances. The step by step method could not be used however because of the prohibitive amount of time it would require. Results obtained by various simplified methods of computations were compared to results of step by step solutions and the method described in the following paragraphs was adopted as most nearly approximating results obtainable by step by step solutions. Subsequently numerous solutions were made with the following values varied over the indicated range.

1. Character of load from 100 per cent lighting to 100 per cent induction motor load.
2. Size of load from 250,000 kw to 1,000,000 kw.
3. Location of fault at Boulder Dam power house and at the load.

From the preliminary studies and the above mentioned series of solutions a load of 600,000 kw was assumed, composed of 60 per cent induction motors and 40 per cent of resistance load. Faults occurring at the Boulder Dam end of the lines gave lower power limits and in the subsequent studies were applied at that point. Solutions made after the above assumptions were established involved the variation of the following items:

1. Character of fault.
2. Design factors of the transmission system.

It is believed that preliminary studies as described above should be made before a simplified method of computation is decided upon. A description of the method used by the authors follows.

The system to be investigated is first represented by a network of resistances and impedances.[4] (See Fig. 5.) The machines are represented by a constant voltage and an impedance in series with it. The system then is reduced through a series of paralleling star to delta, and delta to star, transformations to 2 equivalent machines with a shunt between them.

The negative and zero phase sequence impedances of the system to the assumed point of fault then are calculated. The combination of these 2 corresponding to the character of fault is applied as a shunt to the system at the point of fault and the system reduced to a simple circuit of 2 machines and a shunt. (See Fig. 6.) A third simple equivalent circuit of the system is calculated for conditions after the fault has been cleared.

Transient reactances of synchronous machines are used in the network diagram. The induction motor load is represented by a voltage in series with an impedance calculated as follows: The connected (or capacity) motor load is assumed to be $2^1/_2$ times the load carried, then[1]

$$Z_{ind} = \frac{\text{Base kva}}{\text{Connected kva}} \times 3.00$$

Using $\dot{Z}_1$, $\dot{Z}_2$, and $\dot{Z}_3$ (per unit impedances of the simplified equivalent circuits), the following are calculated:

$$\dot{Z}_{12} = \dot{Z}_1 + \dot{Z}_2 + \frac{\dot{Z}_1\dot{Z}_2}{\dot{Z}_3}$$

This is known as the transfer impedance.

$$\dot{Z}_{11} = \dot{Z}_1 + \frac{\dot{Z}_1\dot{Z}_3}{\dot{Z}_2 + \dot{Z}_3}$$

and

$$\dot{Z}_{22} = \dot{Z}_2 + \frac{\dot{Z}_1\dot{Z}_2}{\dot{Z}_1 + \dot{Z}_3}$$

are known as the driving point impedances. Presenting the above operators in polar form,

$Z_{12}\ \angle\theta_{12}$; $Z_{11}\ \angle\theta_{11}$ and $Z_{22}\ \angle\theta_{22}$

calculate the following scalar values

$$Z_{12};\quad \frac{\cos\theta_{11}}{Z_{11}};\quad \frac{\cos\theta_{22}}{Z_{22}}$$

After all these values are obtained for conditions before, during, and after the fault, air gap voltages of the 2 machines must be evaluated for several load conditions (before fault). Voltage is assumed to be

constant at a point of the system, usually at the junction of the shunt and the 2 machines. If this assumption cannot be made the calculations become slightly more complicated. This voltage is assumed in value and taken as a reference vector for all subsequent calculations. The reference voltage determines the current in the

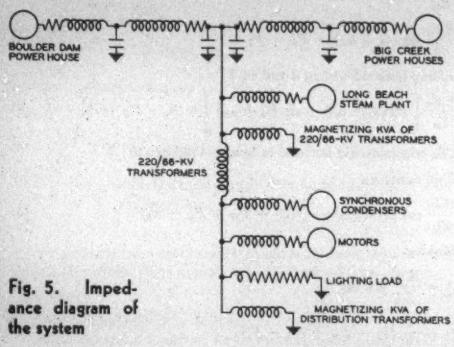


**Fig. 5. Impedance diagram of the system**

shunt. The current from one air gap to the reference point is varied through a desired range and for each value of this current the 2 air gap voltages are calculated; these voltages are $\dot{E}_1$ and $\dot{E}_2$ or $E_1 \angle \delta_{11}$ and $E_2 \angle \delta_{22}$. The angle between the two, $\delta_{11} - \delta_{22} = \delta_0$ is of importance and is expressed in radians. The sin $\delta_0$ and cos $\delta_0$ are tabulated, and sin $\delta_0$ is the value of $D$.

The next step is to determine $A$, $B$, $C$, $D$, $E$, $F$, and $G$ for the given problem.

$A$ is the maximum synchronizing power that can be exchanged by the 2 machines. Its value is $\frac{E_1 E_2}{Z_{12}}$ and it is taken as equal to unity.

$B$ is the maximum synchronizing power than can be exchanged by the 2 machines with the fault applied to the system. It is calculated from the formula

$$B = \frac{Z_{12} \text{ before the fault}}{Z_{12} \text{ during the fault}}$$

$C$ is the maximum synchronizing power that can be exchanged by the 2 machines after the fault has been cleared and is

$$C = \frac{Z_{12} \text{ before the fault}}{Z_{12} \text{ after the fault}}$$

$D$ is the power driving the machines apart before the fault and is equal to sin $\delta_0$.

$E$ is the power driving the machines apart during the fault and is calculated from the formula

$$E = \frac{E_1}{E_2} K_1 - \frac{E_2}{E_1} K_2 + D$$

where

$$K_1 = \frac{Z_{12} \text{ before the fault}}{2} \times \left[ \frac{\cos \theta_{11}}{Z_{11}} \text{ before the fault} - \frac{\cos \theta_{11}}{Z_{11}} \text{ during the fault} \right]$$

and

$$K_2 = \frac{Z_{12} \text{ before the fault}}{2} \times \left[ \frac{\cos \theta_{22}}{Z_{22}} \text{ before the fault} - \frac{\cos \theta_{22}}{Z_{22}} \text{ during the fault} \right]$$

$F$ is the power driving the machines apart after the fault has been cleared and is calculated from the same formula as $E$ except that the values of $\frac{\cos \theta_{11}}{Z_{11}}$ and $\frac{\cos \theta_{22}}{Z_{22}}$ during the fault are replaced by their values after the fault.

$G$ is a numerical value without physical meaning which is used in the solution for the critical angle of clearing $\delta_1$. The calculation of $G$ follows:

$$\delta_2 = \pi - \sin^{-1} \frac{F}{C} \text{ radians}$$

$$G = F\delta_2 \text{ (radians)} - E\delta_0 \text{ (radians)} + C \cos \delta_2 - B \cos \delta_0$$

After $G$ has been determined the critical angle of clearing is found from the following equation:

$$\cos \delta_1 = \frac{E - F}{C - B} \delta_1 + \frac{G}{C - B}$$

where $\delta_1$ is in radians.

The solution from this point on is based on the use of the M.I.T. integraph curves.[2] Four values must be determined to use these integraph curves, namely:

1. $\delta_1$ as evaluated above but expressed in degrees.
2. Sin $\delta_0$ as used in above solution.
3. $\frac{E}{B}$
4. $K$, the time constant.

$$K = \sqrt{\frac{M_0}{314 \times B}} \quad \text{(50 cycle system)}$$

where

$$M_0 = \frac{M_1 M_2}{M_1 + M_2}$$

and

$$M_1 = \sum \frac{0.462 WR^2 \left(\frac{\text{rpm}}{1000}\right)}{\text{Base kw}}$$

(same formula for $M_2$)

$M_1$ and $M_2$ are summations of inertia constants of all the machinery combined into the 2 equivalent machines.

Then sin $\delta_0$ determines the family of integraph curves to be used. $\frac{E}{B}$ determines the curve to be used and $\delta_1$ determines the ordinate of a point on this curve whose abscissa multiplied by the time constant $K$ gives the critical time of clearing in seconds.

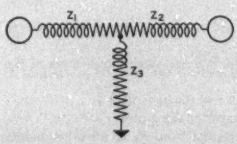


**Fig. 6. Equivalent impedance diagram of the system**

It was found very convenient to plot curves of $\delta_0$, $\frac{E}{B}$ and $\delta_1$ versus load or load current as it eliminated the necessity of interpolating between the M.I.T. integraph curves.[2]

The point of intersection of $\delta_0$ and $\delta_1$ determined the load carried through the disturbance of switching out a section of line (zero time switching). Families of M.I.T. integraph curves were available for sin $\delta_0$ = 0.00, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, and 0.90. Values of $\frac{E}{B}$ and $\delta_1$ corresponding to $\delta_0$ whose sines are

among those tabulated above can be obtained from the curves plotted, and time obtained from the integraph curves with considerable accuracy.

## Appendix II—Derivation of Formula for Solving the Critical Angle of Clearing

As stated in numerous articles, and particularly in articles by Park and Bancker,[1] and Summers and McClure,[2] the system will be stable when the 2 shaded areas of Fig. 7 are equal.

The equations of the 2 areas are

$$A' = \int_{\delta_0}^{\delta_1} E\,d\delta - \int_{\delta_0}^{\delta_1} B \sin\delta\, d\delta$$

$$A'' = \int_{\delta_1}^{\delta_2} C \sin\delta\, d\delta - \int_{\delta_1}^{\delta_2} F\, d\delta$$

integrating and equating the right sides

$$E(\delta_1 - \delta_0) - B(\cos\delta_2 - \cos\delta_1) = C(\cos\delta_1 - \cos\delta_2) - F(\delta_2 - \delta_1)$$

expanding

$$E\delta_1 - E\delta_0 - B\cos\delta_2 + B\cos\delta_1 = C\cos\delta_1 - C\cos\delta_2 - F\delta_2 + F\delta_1$$

rearranging

$$C\cos\delta_1 - B\cos\delta_1 = E\delta_1 - F\delta_1 + F\delta_2 - E\delta_0 - B\cos\delta_2 + C\cos\delta_2$$

taking

$$F\delta_2 - E\delta_0 - B\cos\delta_2 + C\cos\delta_2 = G$$

and substituting

$$C\cos\delta_1 - B\cos\delta_1 = E\delta_1 - F\delta_1 + G$$

factoring

$$(C - B)\cos\delta_1 = (E - F)\,\delta_1 + G$$

dividing by $(C - B)$

$$\cos\delta_1 = \frac{E - F}{C - B}\delta_1 + \frac{G}{C - B}$$

In this equation $\delta_1$ is expressed in radians. To convert the equation to degrees

$$\cos\delta_1 = 0.0174\,\frac{E - F}{C - B}\delta_1 + \frac{G}{C - B}$$

## Appendix III—Mathematical Treatment of the Method

Let:

$E_1$ be the (air gap) voltage at the end of one impedance of the system ($\dot{Z}_1$)

$E_2$ be the (air gap) voltage at the end of the second impedance of the system ($\dot{Z}_2$)

$P_1$ be the air gap input to the stator of the generator

$P_2$ be the air gap output of the stator of the motor

$Z_{11}$ the series impedance of the system from $E_1$ to all other points combined

$Z_{22}$ be the series impedance of the system from $E$ to all other points combined

$Z_{12}$ be the transfer impedance from $E_1$ to $E_2$

$$\dot{Z}_{12} = \dot{Z}_1 + \dot{Z}_2 + \frac{\dot{Z}_1\dot{Z}_2}{\dot{Z}_3}$$

$\delta_0$ is the initial angle between $\dot{E}_1$ and $\dot{E}_2$

$P_{11}$ is power consumed in impedance $Z_{11}$ due to voltage $E_1$

$P_{22}$ is power consumed in impedance $Z_{22}$ due to voltage $E_2$

$\alpha_{12}$ = angle equal to 90 deg less the angle of transfer impedance $\dot{Z}_{12}$

Then, for a stare of equilibrium,

$$P_1 = P_{11} + \frac{E_1E_2}{Z_{12}}\sin(\delta_0 - \alpha_{12}) \quad (1)$$

$$P_2 = P_{22} - \frac{E_1E_2}{Z_{12}}\sin(\delta_0 + \alpha_{12}) \quad (2)$$

$$\frac{E_1E_2}{Z_{12}}\sin(\delta_0 - \alpha_{12}) = P_1 - P_{11} \quad (3)$$

$$\frac{E_1E_2}{Z_{12}}\sin(\delta_0 + \alpha_{12}) = P_{22} - P_2 \quad (4)$$

adding both sides of eq 3 and eq 4

$$\frac{E_1E_2}{Z_{12}}\left[\sin(\delta_0 - \alpha_{12}) + \sin(\delta_0 + \alpha_{12})\right] = P_1 - P_{11} + P_{22} - P_2 \quad (5)$$

The trigonometric function in brackets reduces to

$$2\sin\delta_0\cos\alpha_{12}$$

$$\frac{E_1E_2}{Z_{12}}\cos\alpha_{12}\sin\delta_0 = \frac{1}{2}(P_1 - P_{11} + P_{22} - P_2) \quad (6)$$

Making

$$A = \frac{E_1E_2}{Z_{12}}\cos\alpha_{12} \quad (7)$$

$$D = \frac{1}{2}(P_1 - P_{11} + P_{22} - P_2) \quad (8)$$

we have:

$$A\sin\delta_0 = D \quad (9)$$

This represents the point of intersection of curve $A\sin\delta$ and line $D$ plotted in the torque-angle coördinates.

When a fault (an impedance to ground) is applied suddenly to the system at some point $\dot{Z}_{12}$, $\dot{Z}_{11}$ and $\dot{Z}_{22}$ assume new values giving new values to $\frac{E_1E_2}{Z_{12}}\cos\alpha_{12}$, $P_{11}$ and $P_{22}$ making

$$B = \frac{E_1E_2}{Z_{12}}\cos\alpha_{12} \quad (10)$$

$$E = \frac{1}{2}(P_1 - P_{11} + P_{22} - P_2) \quad (11)$$

eq 9 does not hold. Equilibrium is disturbed and we can write

$$B\sin\delta = E \pm P_b \quad (12)$$

where the physical meaning of $P_b$ is excess or deficiency in synchronizing power over the driving apart power. This power, when negative, is stored as kinetic energy in the mass of the machines, causing relative acceleration, and when positive is taken out of the mass of the machines, causing relative deceleration. By definition "the change in kinetic energy of a body equals the work done on the body" and "work done by a torque acting through an angle is proportional to both."

$$W_b = \int_{\delta_0}^{\delta_1} T_b\,d\delta \quad (13)$$

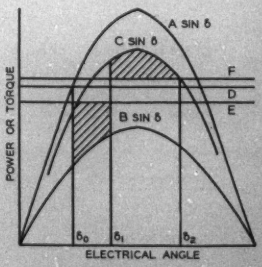


**Fig. 7. Power or torque-angle diagram**

On the torque angle diagram this is represented by an area between $B \sin \delta$ and $E$ (Fig. 7).

When the fault is removed by switching out a faulty section, $E_1$ and $E_2$ are at some angle $\delta_1$ out of equilibrium, and $\dot{Z}_{12}$, $\dot{Z}_{11}$ and $\dot{Z}_{22}$ assume new values. Two new values are then introduced.

$$C = \frac{E_1 E_2}{Z_{12}} \cos \alpha_{12} \tag{14}$$

$$F = \frac{1}{2}(P_1 - P_{11} + P_{22} - P_2) \tag{15}$$

and new resultant equations

$$C \sin \delta = F = P_e \tag{16}$$

$$W_e = \int_{\delta_1}^{\delta_2} T_e \, d\delta \tag{17}$$

$W_b$ is represented on the torque-angle diagram by an area between $C \sin \delta$ and $F$. The equation

$$W_e = W_a \tag{18}$$

is the equal area criterion of stability[1] and is the basis of the method used in this study. The limiting angle $\delta_2$ is found from relation

$$C \sin \delta_2 = F \tag{19}$$

The solution for angle $\delta_1$ is described in Appendix II.

The fundamental equation for a rotating body acted upon by 2 torques is

$$T_1 + T_2 = J_m \alpha = J_m \frac{dw}{dt} = J_m \frac{d^2\theta}{dt^2} \tag{20}$$

where $J_m$ is mass moment of inertia, $\alpha$ is acceleration in radians per second per second, $W$ speed in radians per second, $\theta$ angle of displacement and $t$ = time. Using the nomenclature of this paper

$$P_a + P_d = \frac{M}{2\pi f} \frac{d^2\delta}{dt^2} \tag{21}$$

with proper substitutions, eq 12 becomes

$$-B \sin \delta + E = \frac{M_s}{2\pi f} \frac{d^2\delta}{dt^2} \tag{22}$$

$$\tau = t \sqrt{\frac{2\pi f}{M_s} B} \tag{23}$$

we have

$$\frac{d^2\delta}{d\tau^2} + \sin \delta = \frac{E}{B} \tag{24}$$

Solutions of this equation are available in the form of integraph curves for several values of initial sin $\delta_0$. The integraph curves present angle $\delta$ plotted against $\tau$. The use of these curves is explained in Appendix I.

## Appendix IV—Approximation Introduced by the Use of $M_s = \frac{M_1 M_2}{M_1 + M_2}$

Applying the torque equation to the 2 machines, we have

$$\frac{E_1 E_2}{Z_{12}} \sin (\delta - \alpha_{12}) - P_1 + P_{11} = -\frac{M_1}{2\pi f} \frac{d^2\delta_G}{dt^2} \tag{1}$$

$$\frac{E_1 E_2}{Z_{12}} \sin (\delta + \alpha_{12}) + P_2 - P_{22} = +\frac{M_2}{2\pi f} \frac{d^2\delta_m}{dt^2} \tag{2}$$

Dividing through by inertia constants

$$\frac{2\pi f E_1 E_2}{M_1 Z_{12}} \sin (\delta - \alpha_{12}) - \frac{2\pi f (P_1 - P_{11})}{M_1} = -\frac{d^2\delta_G}{dt^2} \tag{3}$$

$$\frac{2\pi f E_1 E_2}{M_2 Z_{12}} \sin (\delta + \alpha_{12}) - \frac{2\pi f (P_{22} - P_2)}{M_2} = +\frac{d^2\delta_m}{dt^2} \tag{4}$$

we know that

$$-\delta_G + \delta_m = -\delta \tag{5}$$

hence

$$-\frac{d^2\delta_G}{dt^2} + \frac{d^2\delta_m}{dt^2} = -\frac{d^2\delta}{dt^2} \tag{6}$$

Adding up both sides of eq 3 and eq 4

$$\frac{2\pi f}{M_1} \times \frac{E_1 E_2}{Z_{12}} \sin(\delta - \alpha_{12}) + \frac{2\pi f}{M_2} \frac{E_1 E_2}{Z_{12}} \sin (\delta + \alpha_{12}) - \frac{2\pi f (P_1 - P_{11})}{M_1} - \frac{2\pi f (P_{22} - P_2)}{M_2} = -\frac{d^2\delta}{dt^2} \tag{7}$$

Multiplying through by $\frac{M_1 M_2}{2\pi f}$ we have

$$M_2 \frac{E_1 E_2}{Z_{12}} \sin (\delta - \alpha_{12}) + M_1 \frac{E_1 E_2}{Z_{12}} \sin (\delta + \alpha_{12}) - M_2 (P_1 - P_{11}) - M_1 (P_{22} - P_2) = -\frac{M_1 M_2}{2\pi f} \frac{d^2\delta}{dt^2} \tag{8}$$

The first 2 terms can be expanded

$$M_2 \frac{E_1 E_2}{Z_{12}} \sin \delta \cos \alpha_{12} - M_2 \frac{E_1 E_2}{Z_{12}} \cos \delta \sin \alpha_{12} + M_1 \frac{E_1 E_2}{Z_{12}} \sin \delta \cos \alpha_{12} + M_1 \frac{E_1 E_2}{Z_{12}} \cos \delta \sin \alpha_{12} = (M_1 + M_2) \frac{E_1 E_2}{Z_{12}} \cos \alpha_{12} \sin \delta + (M_1 - M_2) \frac{E_1 E_2}{Z_{12}} \cos \delta \sin \alpha_{12} \tag{9}$$

The first approximation is introduced by neglecting the last term of the above equation. $\alpha_{12}$ is usually quite small and so is sin $\alpha_{12}$. The second approximation is introduced when $(P_1 - P_{11})$ and $(P_{22} - P_2)$ are replaced by $\frac{1}{2}(P_1 - P_{11} + P_{22} - P_2)$ in eq 8.

$$(M_1 + M_2) \frac{E_1 E_2}{Z_{12}} \cos \alpha_{12} \sin \delta - (M_1 + M_2) \frac{1}{2}(P_1 - P_{11} + P_{22} - P_2) = -\frac{M_1 M_2}{2\pi f} \frac{d^2\delta}{dt^2} \tag{10}$$

and dividing through by $(M_1 + M_2)$

$$\frac{E_1 E_2}{Z_{12}} \cos \alpha_{12} \sin \delta - \frac{1}{2}(P_1 - P_{11} + P_{22} - P_2) = -\frac{M_1 M_2}{2\pi f (M_1 + M_2)} \frac{d^2\delta}{dt^2} \tag{11}$$

The above equation is

$$B \sin \delta - E = -\frac{M_s}{2\pi f} \frac{d^2\delta}{dt^2} \tag{12}$$

where

$$M_s = \frac{M_1 M_2}{M_1 + M_2}$$

A numerical example will illustrate the approximation

| | | | |
|---|---|---|---|
| $\alpha_{12}$ | = 1.8 deg | $B$ | = 0.552 |
| $M_1$ | = 4 | $P_1$ | = 0.593 |
| $M_2$ | = 12 | $P_{11}$ | = 0.102 |
| | | $P_{22}$ | = 0.446 |
| $\frac{E_1 E_2}{Z_{12}}$ | = 0.552 | $P_2$ | = 0.053 |
| cos $\alpha_{12}$ | = 0.9995 | | |
| sin $\alpha_{12}$ | = 0.0314 | | |

Approximate equation:

$$0.552 \sin \delta - 0.495 = \frac{-3}{2\pi f} \frac{d^2\delta}{dt^2}$$

Correct equation:

$$0.552 \sin \delta - 0.0087 \cos \delta - 0.493 = \frac{-3}{2\pi f} \frac{d^2\delta}{dt^2}$$

When $(P_1 - P_{11})$ and $(P_{22} - P_2)$ differ considerably it may be desirable to introduce the correction

$$M_s = \frac{M_1 M_2 (B - E)}{M_2 [B - (P_1 - P_{11})] + M_1 [B - (P_{22} - P_2)]} \quad (13)$$

In the approximate equation above $M_s$ would be 2.86 instead of 3.00.

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# Electrical Instruments and Measurements—1932-33

**Several important projects were completed by the Institute's committee on instruments and measurements during the past year as indicated in this report.**

DURING the year 1932–33, the A.I.E.E. committee on instruments and measurements consisted of 23 active members, (see footnote) representing various phases of the electrical industry interested in electrical measurements. The active work of the committee was taken care of through 7 subcommittees organized as follows: instrument transformers, indicating instruments, telemetering, high frequency and sound measurements, temperature measurements, definitions of instruments and testing, and measurement of transformer exciting current. Activities of these subcommittees are summarized in the first portion of this report.

### Instrument Transformers

The subcommittee on instrument transformers was engaged in completing the revision of the Standards for Instrument Transformers. The report made at the October meeting of the committee was accepted and the standard circulated to the members for letter ballot. The result of this ballot indicated that almost all the members were ready to approve the revision with certain minor changes. These are being made, and the revision will be submitted to the A.I.E.E. standards committee for acceptance.

This standard has entailed considerable work on the part of the subcommittee, and led to rather wide discussion of suggested changes in the definition of "phase angle for instrument transformers," which later was taken up by the standards committee.

### Indicating Instruments

The Indicating Instrument Standards No. 33 has been under consideration for revision by this subcommittee. The work was advanced to the stage where a draft was circulated to the membership of the main committee for comment. Based upon the replies, an approved draft now is being prepared and will be ready at an early date for submission to the standards committee.

Recently the American Standards Association appointed a sectional committee to consider the adoption of the Institute standards as an American standard. It is expected that the revised issue will be the basis for discussion by this sectional committee on which the instruments and measurements committee has 3 representatives.

### Telemetering

The subcommittee on telemetering during the past year has devoted its work primarily to following new developments in its field. At the summer convention in 1932, it submitted, in coöperation with the automatic stations committee, "A Report on Telemetering, Supervisory Control, and Associated Communication Circuits" (see Electrical Engineering, v. 51, September 1932, p. 613–20). This report is a comprehensive survey of available systems, and will serve as a standard of reference for several years.

### High Frequency and Sound Measurements

The subcommittee on high frequency and sound measurements is working with a committee appointed by the Standards Committee to draft definitions and standards for sound measurements. Through the efforts of this subcommittee, an informal session on sound measurements was held during the 1933 winter convention, and at the summer convention a paper on that subject was presented representing the progress in that field to date. Activities in the field of high frequency measurement were outlined by this subcommittee, but, because of present conditions, completion of several intended studies was delayed. This work had to do with measurements of resistance, voltage, and current.

### Temperature Measurements

Work planned by this subcommittee included the preparation of a standard code for temperature

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Excerpts from the annual report of the A.I.E.E. committee on instruments and measurements for 1932–33. *Not published in pamphlet form.*

Committee on instruments and measurements, 1932–33: E. J. Rutan, *chairman;* H. C. Koenig, *vice-chairman;* R. T. Pierce, *secretary;* H. S. Baker, R. D. Bean, O. J. Bliss, P. A. Borden, H. B. Brooks, A. L. Cook, E. D. Doyle, W. W. Eberhardt, Marion Eppley, W. N. Goodwin, Jr., I. F. Kinnard, O. A. Knopp, A. E. Knowlton, W. B. Kouwenhoven, F. A. Laws, E. S. Lee, J. B. Lunsford, Paul MacGahan, W. J. Shackleton, and H. L. Thomson.

measurements. Considerable data have been collected, but will require additional study before the preparation of a tentative draft. It is expected, however, that during the coming year this work will be completed and made available for the Institute.

### Definitions of Instruments and Testing

As reported last year, the work of this subcommittee was completed and these definitions submitted to the working committee. Since that time, the definitions have been published. It has been recommended recently that several terms that have not been defined in these definitions be included; these definitions are to be presented to the working committee for their consideration.

### Measurement of Transformer Exciting Current

The transformer subcommittee of the electrical machinery committee transmitted to the instruments and measurements committee a request to prepare suitable methods for measuring exciting current of transformers when excited with other than pure sine waves. The subcommittee on measurement of transformer exciting current undertook this work and reported a suitable procedure at the April meeting; this was adopted and has been forwarded to the transformer subcommittee for their use. The report includes a recommended method, but also discusses other methods available and their limitations.

### Review of A.S.M.E. Code

In addition to the foregoing work, the instruments and measurements committee reviewed a test code prepared by the American Society of Mechanical Engineers dealing with electrical instruments. This work was speedily completed and approval given so that A.S.M.E. could undertake publication of this material, which already was in final proof form. It was brought to the attention of the standards committee that it would be desirable to arrange to have the A.S.M.E. refer to A.I.E.E. standards and codes for such information rather than prepare their own publications in this field. It is understood that this suggestion has received favorable consideration.

### Future Work

In addition to the present activities, a suggestion was made that the instruments and measurements committee undertake the study of methods for surge voltage measurements. This suggestion is an outcome of papers presented under the auspices of the committee at the 1933 winter convention, and the ensuing discussion.

Considering conditions that exist, the work of the instruments and measurements committee has shown hardly any curtailment. It is felt that such activity is a result of the efforts of the vice-chairman, the secretary and the chairmen of the subcommittees who have carried on their work so that several important projects have been completed. There is no doubt that the present membership of this committee is an active one and of sufficiently broad connections so as to be productive of further good work in the future.

# Electrical Communication—1932-33

**Some of the outstanding advances in electrical communication during the past year are described briefly in this report by the Institute's committee on communication.**

SOME of the advances in electrical communication were described in detail in technical papers sponsored by the Institute's committee on communication during the past year; others are outlined briefly in this report. Since it is desirable to keep the report short, only a few of the outstanding advances are mentioned.

A new type of telegraph repeater recently has been put into use to facilitate repeating multiplex telegraph channels individually into other multiplex circuits, which has the advantage of not requiring the maintenance of synchronism between the multiplex circuits concerned. An important application is a direct Montreal-London connection involving a Montreal-New York multiplex circuit and the 8-channel New York-London loaded cable. A multiplex telegraph channel concentration arrangement has been developed and installed in recent new telegraph offices. Operating positions, concentrated in the operating room, are connected through a switchboard located in the operating room to multiplex distributors grouped in the testing and regulating department. Reassignments of operating positions to meet changes in load readily are made, and important economies are effected in equipment, space, personnel, and other costs. There also has been developed a signal distortion indicating device for use in regulating and adjusting start-stop printer circuits without interrupting traffic. The device is of a stroboscopic nature and it indicates the effect of transmitter speed irregularities, relay bias, and other causes of signal distortion.

Successful experiments were carried out with a new

Excerpts from the annual report of the A.I.E.E. committee on communication for 1932-33. *Not published in pamphlet form.*

Committee on communication, 1932-33: H. S. Osborne, *chairman;* E. J. O'Connell, *secretary;* H. M. Bascom, W. H. Capen, A. A. Clokey, J. O'R. Coleman, C. F. Craig, R. D. Evans, W. L. Everitt, I. C. Forshee, C. M. Jansky, Jr., T. Johnson, Jr., G. M. Keenan, G. A. Kositzky, C. J. Larsen, J. R. MacGregor, John Mills, J. W. Milnor, C. W. Mitchell, P. H. Patton, F. H. Pumphrey, F. A. Raymond, H. A. Shepard, E. R. Shute, A. L. Stadermann, C. H. Taylor, H. M. Turner, and F. A. Wolff.

method of superimposing telegraphy on telephony on the Madrid-Buenos Aires radiotelephone link. By means of inverters and spreaders in the telephone channel, and by appropriate allocation of frequencies, telegraphy at 125 words per minute was possible at the same time as speech.

Development of a new 18-channel voice frequency carrier telegraph system was completed and 3 of these systems have been installed in England. In this country trials of a 24-channel voice frequency system have shown that such a system is satisfactory from a technical standpoint.

During the year the British Post Office introduced person-to-person written communication service. This was called "Telex" service and is furnished to a subscriber over his telephone loop; he may talk or typewrite over the same connection, although not simultaneously.

In the United States an interesting development of teletypewriter service associated with telephone lines is the application of these devices to the direct printing of weather maps over extensive telephone plant networks for use in connection with airplane service. Outline maps are placed in the machines at airports connected to this system, and symbols transmitted from the U.S. Weather Bureau in Washington are recorded simultaneously at all of the stations in the group covered by one section of the weather map. These symbols show all of the regular items of the weather map.

To meet further demands for high-grade and economical circuits in cable, considerable development work has been carried out, including an extensive experimental installation of telephone carrier applied to a 25-mile loop of underground cable. A carrier system design has been achieved in which the difficulties due to enormously increased attenuation and increased tendency to crosstalk are surmounted. Sufficient work has been done to demonstrate that the system is entirely practicable. Carrier applied to cables offers important possibilities of economy, particularly for routes carrying heavy traffic. Telephone transmission improvement also is effected; this is particularly important for long circuits where, with present cable methods, transmission delays introduce difficulties. A paper on this subject was presented at the 1933 summer convention (see "Communication by Carrier in Cable" by A. B. Clark and B. W. Kendall, ELECTRICAL ENGINEERING, v. 52, July 1933, p. 477–81).

An interesting application of this method of cable transmission was made in recent experiments in high grade transmission of orchestra music. The new transmission system including pick-up arrangements, transmission lines, amplifiers, and loud speakers, represents a marked advance over systems previously developed for the transmission of music, in the following respects:

1. "Auditory perspective," that is for example, the reproduced sounds of different instruments of an orchestra appear to come from different parts of the stage, corresponding to the actual relative location of these instruments at the transmitting point.

2. Frequency spectrum covering practically the entire audible range, that is, from 35 to 15,000 cycles.

3. An intensity range corresponding to short-time energy differences between strong and weak passages of 10 million to one, making possible a volume of reproduced sound at least 10 times as great as that produced by the orchestra itself.

4. Increased control over the volume of sound at the receiving point and the relative loudness of various parts of the orchestra, making possible an enhancement of the musical effects over that produced by the orchestra itself.

Public demonstration of this system was made in Washington on the evening of April 27, 1933, with the Philadelphia Symphony Orchestra playing in Philadelphia. Transmission over the long distance circuits was so excellent that there was no appreciable difference in the overall characteristics of the system with or without the long distance lines.

The Italy-Sardinia submarine cable, the longest submarine telephone cable in the world (approximately 150 nautical miles) was completed and put in service. This continuously loaded cable, laid between Fiumicino, Italy, and Terra Nova, Sardinia, provides a single 2-way circuit on which are operated 1 composited duplex telegraph channel and 1 telephone channel; it is designed for the addition of a 2-way carrier telegraph channel. Because of the long length and consequent high attenuation, special methods have to be used to achieve a high singing point in order to operate the circuit on a 2-wire basis.

Multi-party toll service or toll conference service was made available throughout the United States to a large portion of the telephone stations. Connections affording 2-way communication among not more than 6 parties are furnished immediately at the subscriber's request. For some of the more distant connections and those involving more than 6 parties, the service is being given on appointment where practicable.

A telephone cable between Kansas City, Mo., and Dallas, Tex., was completed during 1932, thus connecting Dallas and other Texas points into the toll cable network which now provides a storm-proof system covering most of the eastern half of the United States. Among other circuits this cable includes direct New York City-Dallas circuits, 1,850 miles in length, which are the longest direct all-cable telephone circuits in the world.

Micro-ray radiotelephone equipment is being built for the British Air Ministry and will be erected at Lympne, England. This will operate on a wave length of 15 cm in conjunction with similar equipment to be situated at St. Inglevert aerodrome, near Calais, France. This is the first commercial application of the micro-ray system. An interesting feature of this new service will be the use of teleprinters for transmitting and receiving messages.

A new technique of grinding tourmaline crystals makes it possible to produce such crystals commercially for wave lengths as low as 5 m.

Several new types of direction finders for ships have been developed, one of which incorporates an automatic indicator showing the bearings of the station at any time. The new direction finders need no correction as the quadrantal error is eliminated permanently by suitable design of the loop and antenna systems.

A considerable number of aircraft, formerly having only 1-way equipment for receiving beacon signals and weather reports, now have been equipped with 2-

way radiotelephone equipment. Experiments on blind flying by means of directional radio signals have been continued with excellent results. By means of equipment on the aeroplane which indicates very accurately the landing runway and the altitude of the plane, it is now possible to land safely when the pilot is unable to see the ground.

The use of high power vacuum tube amplifiers in sound recording has made possible the development of an improved vertically cut record. Less inherent distortion, greater volume range, and a playing time of 15 to 20 min on a 12-in. record are possible with the new process. By using a reproducing stylus of very light weight, the records last for several thousand playings with no noticeable deterioration.

Experiments with the use of highly accurate frequency standards in connection with the control of power system operations are being made. The frequency standards, having their source in quartz crystal oscillators, are supplied to the control mechanism over telephone circuits.

In its a-c electrified territory one of the railroads has installed a neutralizing wire in its telephone conduit system connected to the track and substations through impedance bonds, and an aerial tape armored telephone cable for the mitigation of inductive interference; the results are highly satisfactory.

In nearly all municipal fire alarm systems that have been remodeled during the past year, copper oxide rectifiers, supplied with alternating current at 110-volts, have been used as sources of power.

# Electricity in Transportation—1932-33

**During the past year several notable applications of electricity have been made in transportation both on land and on sea. These are outlined briefly in this report by the Institute's committee on transportation.**

THE PRINCIPAL DEVELOPMENT of the past year in the field of heavy electric traction was the progress made by the Pennsylvania Railroad on its project for complete electrification of its lines between New York City and Washington, D. C. Electrical operation of passenger trains between New York and Philadelphia was begun on January 16, 1933, and since then has been extended to cover all New York to Philadelphia passenger service and also the New York to Washington passenger service as far as Wilmington, Del. Power is supplied from the railroad company's generating station in Long Island City, N. Y., and from 2 frequency converter stations of the Philadelphia Electric Company. In the more recent of these stations the converter units are not placed in a building but are protected by steel housings, the installation being similar to, but larger than, that made by the same company to supply the Reading Company's electrification. Single-phase transmission lines, generally 4 in number, suspended from tall H-section poles (which also support the catenary system) carry this 25-cycle power at 132-kv to step-down transformer substations, spaced from 7 to 10 miles apart along the right-of-way, from which the 11-kv contact wires are energized. Between New York and Wilmington and in the Philadelphia suburban zone approximately 72 locomotives and 382 multiple unit motor cars were in service on April 1, 1933. Including the Long Island lines, the Pennsylvania system now has 1,450 miles of electrically operated track.

The Reading Company completed the electrification of its Norristown and Chestnut Hill branches, and began electrical operation over them on February 1, 1933. Thus 46 track miles, on 22 miles of route, have been added to the mileage over which electrical operation was begun in 1931. A description of this electrification is contained in a paper at the 1932 A.I.E.E. Pacific Coast convention (see "The Reading Railroad's Suburban Electrification" by G. I. Wright, ELECTRICAL ENGINEERING, v. 52, March 1933, p. 155–61).

The New York Central Railroad continued work on its "West Side Improvements," to remove its freight tracks from the streets of New York City south of 60th Street.

The Atchison, Topeka, and Santa Fe Railroad placed in service a 900-hp articulated gasoline-electric rail car, made up of 2 sections, the forward part carrying the engine, generator, and other equipment, while the other is divided into passenger and baggage compartments. This is the most powerful unit of this type that has been built. It can reach a speed of 80 mph on level tangent track and has sufficient power to handle 4 trailing coaches.

An oil-electric rail car of 600 hp has been built, which is capable of a speed of 80 mph and can haul 3 coaches. It weighs 104 tons and carries 2 300-hp diesel engines with direct-connected generators and 4 traction motors. Manufacturers state that they are now prepared to furnish units of this type with a capacity as high as 1,000 hp.

## URBAN TRANSPORTATION

The New York City Board of Transportation on September 10, 1932, placed in operation the first section of the City's independent subway system, this route being the Eighth Avenue subway, under the westerly side of Manhattan from Fulton Street to 207th Street. During 1933, operation has been extended from Fulton Street, Manhattan, under the

Full text of the annual report of the A.I.E.E. committee on transportation for 1932–33. *Not published in pamphlet form.*

Committee on transportation, 1932–33: E. L. Moreland, *chairman;* H. L. Andrews, Reinier Beeuwkes, A. E. Bettis, H. A. Currie, J. V. B. Duer, H. H. Field, I. W. Fisk, W. A. Giger, K. T. Healy, A. E. Knowlton, H. N. Latey, John Murphy, H. Parodi, R. H. Rice, S. A. Spalding, N. W. Storer, W. M. Vanderaluis, R. P. Winton, Sidney Withington, and G. I. Wright.

East River to Bergen Street, Brooklyn; from 155th Street under Grand Concourse to 205th Street, Bronx; and from 53rd Street under the East River to Roosevelt Avenue, Queensborough.

In Philadelphia additional sections of subway have been placed in operation.

Additional cities have installed trolley buses so that at the close of 1932 it was reported that 280 buses were in operation in 20 cities in the United States (including Manila) over approximately 270 miles of route.

In the continuing work of developing an improved street car design, attention is being given to the elimination of vibration by using resilient wheels and rubber cushioning instead of steel springs in trucks. Another feature being developed is an eddy-current drum-type brake, incorporated in the motor frame, which will use power from the trolley. Hydraulic brakes will supplement these brakes at low speeds or when power is not available.

### Railway Signaling

Signaling construction was much reduced during the past year, attention being given principally to small installations of automatic interlocking and centralized traffic control offering substantial economies in operation.

The first application of centralized traffic control to subway operation is being made in the extension of the Philadelphia system. A central plant at Market Street Station will control all the emergency crossovers on the new line, individual interlocking plants at the crossover locations being unnecessary; this central plant will permit complete control of all switching and traffic movement in its territory, besides furnishing visual indication of the position of trains on an illuminated track model and also making a permanent record of train movements on an automatic graphic recorder. This installation is noteworthy because of the density of traffic involved and because of modifications of design so that alternating current is used exclusively in its operation.

Signal engineering frequently must overcome difficulties presented by new developments in other lines of transportation activity. In the New York terminal zone of the Pennsylvania Railroad some tracks have been equipped with overhead contact system as well as third rail, and now are utilized for both 11-kv single-phase and 600-volt d-c operation. Saturation effects of the direct current in the alternating-current locomotive and substation transformers produce harmonics which interfered with the proper functioning of the 100-cycle track circuits controlling the wayside signals and the locomotive cab signals. This difficulty was overcome by reducing the signal circuit frequency in this area to $91\frac{2}{3}$ cycles and by modifying the design of the cab signal apparatus so that it will function properly at the lower frequency while in the terminal zone and also at 100 cycles on other sections of the road.

Another problem has been presented by the operation of light-weight rail cars, particularly those with rubber-tired or rubber-cushioned wheels, since they do not provide the low resistance connection between the rails required for shunting signal track circuits. This problem has been solved by utilizing a high frequency alternating current which is generated on the car, passed through step-down transformers having low resistance secondary windings, and then applied to the track by means of rail brushes on rubber-tired cars or through collector rings on the steel tires of rubber-cushioned wheels. The alternating current breaks down the high-resistance film on the rail surface and a low resistance path through the transformer secondaries thus is provided for shunting the track circuits.

### Marine Transportation

The most notable application of turbine-electric drive to new ships in the past year was in 6 vessels built for the United Mail Steamship Company (a subsidiary of the United Fruit Company). Each of these has 2 main turbine-generators, each rated at 4,200 kw, 3,500 rpm, 3,150 volts, 3-phase, unity power factor, which furnish power for 2 5,250-hp 125-rpm synchronous propelling motors. Extensive use is made of electricity to operate the auxiliaries in the engine rooms, ventilating fans, etc., throughout the ships, and all of the deck machinery.

The Manhattan, largest liner ever built in America, while driven by geared turbines, has extensive electrification of auxiliary machinery and of miscellaneous services throughout the ship. Four 500-kw turbine-generators supply 383 motors totaling nearly 4,000 hp, in addition to 1,200 fans in living quarters, 8,387 lamps, electric cooking apparatus sufficient for the preparation of 5,000 meals per day, electric clocks, automatic steering, radio, sound motion picture machinery, and many other applications. A 75-kw diesel engine driven unit also is provided for emergency use.

The 3 sister ships of the Matson Line, the Mariposa, the Monterey, and the Lurline, are equipped similarly, each having 4 500-kw turbine-generator sets and one 30-kw diesel engine-generator set, for supplying power apparatus totaling 2,600-kw, and lighting units, 230 kw.

Four new ships built for the Grace Line, to be used in its intercoastal service via the Panama Canal, also have direct geared-turbine drive; but their auxiliaries have been completely electrified, as have those on the so-called "Seatrains" operating between New York City and Havana, Cuba. These vessels, built to transport loaded freight cars, are noteworthy as the first ocean going ships on which a-c motors are used exclusively.

### Vertical Transportation

The central tower of the Rockefeller Center (which is now New York City's largest building) although exceeded in height by the Empire State Building, is equipped with 74 elevators, capable of the highest speed now permitted, namely, 1,200 ft per minute. On part of these elevators an interesting application of photoelectric cells functions to prevent closing the high-speed power-operated doors while a passenger is entering.

# Trolley Wire Lubrication Improved

**Laboratory tests indicate that solid graphitic material for trolley wire lubrication is vastly superior to the graphite-grease now commonly used for that purpose. Tests under practical operating conditions now are being made on a 210-mile electrified portion of the Chicago, Milwaukee, St. Paul, and Pacific Railroad. This paper discusses briefly the general problem of trolley wire lubrication, and presents results of tests on both graphite-grease and solid lubricant.**

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OPERATION of an electrified railroad involves a certain amount of wear on the contact surfaces of the trolley wire and moving current collectors; this wear is dependent largely upon the type and amount of lubrication used on the contact surfaces. Such lubrication, early recognized as an important part of electric railroad operation, is the subject of extensive improvement resulting from an entirely new and different method of applying a solid lubricant. This paper outlines laboratory experiments devised especially for the study of contact surface lubrication, the results obtained, and some of the possibilities of the use of solid lubricant.

Summarizing the conclusions drawn as a result of these experiments, it appears that a solid graphitic material can be used to give effective trolley wire contact lubrication with its attendant reduction in the rate of wear and expense of worn part replacement. Such lubrication promotes longer wire life, which in turn avoids reduced wire cross section and mechanical breakages; it eliminates all forms of grease with its highly undesirable dirt; and current collection conditions are improved to the advantage of reduced communication interference, which ordinarily is caused to a certain extent by sparking at the collector contact.

Due to its very nature and method of application the solid type of lubricant is applied automatically on rough surfaces only, where needed, and all of that part that does wear away is deposited and distributed effectively on the contact surfaces instead of being dropped wastefully on the locomotive equipment as is graphite-grease. The rate of wear of the solid lubricant is sufficiently low to make a single application last several thousand miles; this feature makes road attention unnecessary, thereby forestalling the possibility of electrical injury or death to a man working on a pantograph that accidentally might become energized. When contact lubrication with solid materials wins favor in the field as it has in the laboratory, the possibilities of this method of lubrication will contribute toward the improvement of current collection on electrified railroads throughout the world. The results obtained in these laboratory experiments offered ample justification for an extensive service trial, which is now in progress with a form of this type of lubrication being used, to the exclusion of graphite-grease lubrication, on the 210-mile Coast division of the Chicago, Milwaukee, St. Paul, and Pacific Railroad.

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## General

Ever since the advent of electric traction motors for the operation of trains, cars, and other vehicles that receive electric power through contact with a third rail or trolley wire, the design of current collectors and contact lines has been an important part of all systems of electric traction. Although many different forms of wheels, rollers, bows, and sliders have been in extensive use, the slider type of metal-to-metal contact at present is accepted as the most satisfactory device for current collection on fast, high-powered equipment; the scope of this paper is confined to a discussion of lubrication problems incident to the slider type of current collector.

Trolley wire lubrication is of interest to operators of all electrified railroads on which power is distributed by means of an overhead contact line, and this paper deals with the subject from their point of view. However, the results obtained in these experiments can be interpreted as applying to any exposed, sliding, or moving, metal-to-metal contacts that carry electric current.

A large majority of electric traction systems both in the United States and other countries use an overhead contact system with some form of sliding shoes mounted on bow or pantograph type current collectors which receive power for the operation of traction motors and necessary auxiliary equipment on electric locomotives and motor cars. On all these systems it is desirable for reasons of economy and operating efficiency to reduce the wear on trolley wire and contact strips to a minimum; it is with this object in view that the subject of contact lubrication has been given the consideration outlined herein.

Operating data covering trolley wire wear, contact surface condition, and locomotive movements were obtained from the Chicago, Milwaukee, St. Paul, and Pacific Railroad, which has hundreds of miles of main line road under complete electrical operation over a territory imposing practically every conceivable condition of speed, tonnage, track curvature, grade, temperature, and climate. This system made an excellent field for the study of wire wear and contact surface lubrication.

## Need of Wire Lubrication

Trolley wire, being subject to wear, eventually becomes greatly reduced in cross section, with many attendant disadvantages such as decreased mechanical strength, electrical conductivity, and reliability. The inevitable delays and losses caused by the breakage of worn trolley wire have an economic bearing on the operation of any electrified railroad; and although the cost of such losses cannot be estimated accurately, it is logical to assume that they should not be overlooked. In the United States alone the regular operation of some 3,500 pantograph type current collectors, which slide in contact with trolley wire to the extent of more than 100,000,000 pantograph miles per year, involves a maintenance and replacement expense that depends directly upon the wear and tear on the trolley wire and contact equipment. Effective contact surface lubrication therefore is desirable where it prevents comparatively rapid wear of trolley wire and contact strips.

Wear tests conducted nearly 20 years ago by some of the electric railway equipment manufacturers conclusively demonstrated the value of using some form of contact surface lubricant for the reduction of wire and collector strip wear. When first adopted, the use of graphite-grease (a mixture of flake graphite and motor grease) as a contact lubricant held promise of solving many of the problems surrounding wire wear; however, experience has demonstrated it to be ineffective in many ways as a lubricant principally because of the method of application. It is also very dirty and somewhat hazardous. The graphite-grease adheres to the trolley wire, pantograph parts, and the tops of locomotives to such an extent that it interferes with workmen in their regular duties. Linemen, for example, find that greased trolley wire is extremely messy and awkward to handle when contact line repairs are made. The grease also adheres so tenaciously to the upper parts of the locomotives that cleaning and scrubbing the fouled parts is a problem of no small significance. When the grease drops on the walkways of the locomotive roofs it also greatly increases the possibility of a man slipping and falling from a dangerous height to the ground.

The graphite-grease normally is applied between 2 copper contact strips of a pantograph shoe as shown in Fig. 1; however its effectiveness is dependent upon the distance between the section of line in question and the locomotive terminal, where fresh lubricant is applied. Where the grease is rubbed on the wire, an effective lubrication results; but where the wire is remote from a locomotive terminal, the lubricant actually applied to the trolley wire is reduced in quality and quantity to a marked extent. In many cases the lubricant used with the graphite-grease filled shoe breaks down to the point where a layer of worn copper dust and grit covers the graphite-grease, stopping all lubrication.

The effect of terminal distance on wire wear is indicated in Fig. 2, which is derived from a study of over 500 wire measurements secured on the Rocky Mountain division of the Milwaukee railroad. The contact line consists of a pair of 0000 B. & S. gage copper trolley wires; d-c power is distributed over them at a potential of 3,000 volts. The section of line shown on the chart is part of a 440-mile division, which has locomotive terminals at the middle and at each end. Measurements showed that the trolley wire wear was a minimum at the terminals and a maximum midway between them.

Wear over curved track is somewhat less than that over tangent track due to the use of a chord type of overhead contact line, which has a tendency to sweep transversely across the moving pantograph shoes as a locomotive moves along the line. The graphite-grease which is usually clean and abundant near the extreme ends of the pantograph shoes thus is spread on the contact surface of the trolley wire over curved track resulting in a more effective lubrication at such points. The lubricant at the extreme ends of the shoes rarely is touched while a locomotive is on tangent track unless the trolley wire is out of alignment or the locomotive is subject to violent rolling.

Wire measurements taken on the trolley wire always should be supplemented by a lubrication mark. This mark is made by holding a page of an open notebook face upward against the contact surface of the trolley wire and moving it along the wire for a few inches. Near the locomotive terminals the abundant supply of graphite-grease on the wire transfers patches of thick black grease to the book, but at points a few miles away from the terminals a sharp black impression is left on the page of the book indicating that the contact surface of the wire is covered with a lubricating film of moderate thickness. At slightly greater distances from the terminals the clean white notebook page slides quite easily along the wire and picks up very little coloring from the smooth, glossy contact surface. Measure-

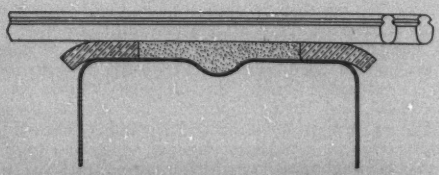

Fig. 1. Section of slider type pantograph shoe under 2 trolley wires. Space for lubricant is provided between the 2 contact strips

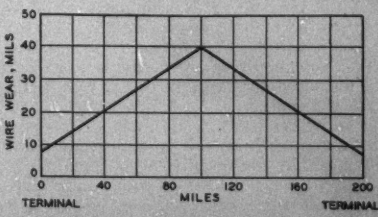


Fig. 2. Trolley wire wear on tangent track with graphite-grease applied at terminals; 50,000 locomotive passes

Fig. 3 (left). View of testing machine showing end of collector shoe and slider carriage; graphite-grease lubrication

Fig. 4 (right). General view of testing machine showing in the foreground the mechanism used to impart radial motion to slider carriage for simulating locomotive "roll"

ments taken at points midway between locomotive terminals showed the maximum wear on the wire; and when attempts were made to record lubrication marks in the notebook, the wire surface was found in some cases to be sufficiently rough to tear the surface of the page. Thus there were extreme conditions, ranging from an oversupply to a complete absence of lubricant within a distance of 100 miles.

Such observations indicated that there was much room for improvement in wire contact surface lubrication as the graphite-grease was applied wastefully without giving satisfactory results and it was therefore undesirable lubricant for the purpose.

## Laboratory Test Machine Construction

Any experimental work on contact surface lubrication that could be accomplished in actual operation on a railroad, even on a large scale, might require a period of years to produce information of definite value. It appeared, therefore, that some other means, which would produce greatly accelerated results, would be required. The nature and importance of the problem warranted a sufficiently thorough study to justify the construction of a special test machine by means of which accelerated tests could be made under close observation and control. The relative effectiveness that different types of lubricant would give in laboratory experiments would provide an indication of the relative effectiveness and performance that might be expected of these lubricants under actual line conditions.

Such a test machine was designed and built at the University of Washington electrical engineering laboratory during the fall of 1931 and was in operation late in December of that year. It is illustrated in Figs. 3 and 4. The machine consisted principally of a wheel 46 in. in diameter mounted on a vertical shaft and driven by a d-c motor through a twisted belt. A ring of 0000 trolley wire or $^1/_4$-in. copper bus bar (depending on the tests) was mounted on the outer rim; when in operation this copper ring turned with the wheel and ran over the surface of a collector shoe which was spring mounted and held against the ring with a constant pressure. The standard upward pressure of $7^1/_2$ lb per shoe per wire, corresponding to 30 lb per pantograph, was adhered to throughout the experiments; likewise the standard current density of 200 amp per shoe per wire, or 800 amp per pantograph, was the same as that used on the Milwaukee 250-ton d-c locomotives operating on 3,000 volts.

For supplying the contact current, a motor-generator set was used; this was a 300-amp General Electric are welding unit which furnished 200 amp at 6 volts continuously. A current of 200 amp was circulated from the positive generator terminal through a cable to brush holders bearing on the pulley of the wheel, into the wheel shaft and out through the metal spokes to the copper contact ring; from this ring it passed through the sliding contact to the spring mounted collector shoe and thence through a cable back to the negative generator terminal. The current path was practically a short circuit on the generator, which, however, had a very rapid voltage recovery and formed a violent arc whenever there was any separation at the sliding contact. The arc-welding type of generator selected for this purpose served to duplicate line conditions closely, as any amount of contact separation due to rough surfaces or skipping of the current collector shoe caused destructive arcing similar to that sometimes experienced in railroad operation. This characteristic would not have been available with a simple low-voltage electro-plating type of shunt generator.

The current collector shoe was mounted on a carriage that moved on a pair of slide rods with an irregular transverse motion maintained by a small motor driven gear mechanism to simulate the rolling action of a moving locomotive. The gear mechanism used to sweep the collector shoe carriage back and forth is shown in Fig. 4.

## Preliminary Operation of the Apparatus

A fair amount of experimenting was done on the test wheel before the regular schedule of testing was adopted, and some minor changes were made to facilitate the regular work and improve the performance of the machine. Probably the most important refinement was the turning or machining of the contact surface of the wire ring to remove minute waves in the wire, to prevent skipping of the collector shoe at high speed, and to improve generally the smoothness of operation of the test machine. During the preliminary runs with graphite-grease as a lubricant,

Fig. 5. Contact surface of trolley wire tested without lubrication

numerous failures of the lubricating film left the machine running without lubrication. The contact friction under these conditions was extremely high, and the current collection was subject to arcing and flashing which soon destroyed the contact surfaces. A view of the wire contact surface after a short run without lubrication is shown in Fig. 5, which clearly illustrates the rough nature of the contact surface and the burning and scoring caused by arcing and excessive friction. Conditions similar to those obtained without lubrication are brought about by failure of the graphite-grease lubricating film, an example of which is illustrated in Fig. 6. Such evidence as that exhibited in Figs. 5 and 6 bears out the contention that contact surface lubrication is highly important. Operation of any electric traction system without effective contact lubrication obviously would be destructive, unsatisfactory, and costly.

## Test of Graphite-Grease

With graphite-grease used as a lubricant several test runs were made with the machine for the purpose of comparing the laboratory data with that obtained in actual railroad operation. Conditions maintained and observed during this test were as follows:

*Constants*

| | |
|---|---|
| Rim speed | 40 mph |
| Contact current | 200 amp continuously |
| Roll frequency | 24 cycles per minute |
| Collector shoe pressure | 7½ lb |

Periodical application of graphite-grease was made every 3 hours, which was the time required for the passage of 119 miles of wheel ring over the collector shoe. This is the practice followed in railroad operation.

*Observations*

Wheel turns or "passes"
Wire thickness
Readings taken every 50,000 turns (every 3 hr)

A turn is the same as a "pass" as used in railroad parlance and means that every time the wheel makes one complete revolution, any one point on the rim of the wheel makes a pass over the collector shoe. This would have the same effect as that which would be obtained if the wheel remained stationary and a locomotive collector shoe passed under it.

Wear on a trolley wire is essentially a function of the number of contact shoes or locomotives passing under it and for that reason wheel turns or revolutions are directly comparable to locomotive passes. Wear measurements on the current collector shoe were not recorded as they depend upon the miles of wire passed over the shoe; for a laboratory machine like the one used in these tests, this mileage is small and of little consequence compared with the passes made by any one point on the rim of the wheel.

Fig. 6. Contact surface of current collector shoe tested with graphite-grease between copper contact strips. Part of lubricant whipped out of shoe and surface of remaining lubricant became covered with copper dust

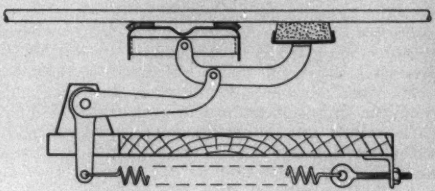

Fig. 7. Diagram of mechanical rigging used with auxiliary lubricator shoe, the lubricator shoe is at the right

Results obtained during the graphite-grease tests were comparable to existing line conditions and indicated that the test machine could be used satisfactorily for determining comparative data on the effectiveness of solid and graphite-grease lubricants.

## Test of Solid Graphitic Material

The collector shoe carriage was rebuilt to accommodate an auxiliary lubricator shoe for solid graphitic lubricant and the 2 shoes, current collector, and lubricator, were mounted on 2 levers each pivot-supported on the end near the current collector shoe (see Fig. 7). There was a set of these levers under each end of the 2 shoes. The pivot support was located in such a position that the lubricator shoe pressure against the wire ring was ½ the current collector shoe pressure.

Before the auxiliary shoe with solid lubricant was tested, there was considerable doubt as to the ability of the solid graphitic material to withstand continuous service, for the lubricant was softer than most motor brushes and at times would be subject to severe abrasion and scoring. The first test, however, and all subsequent tests clearly showed the superiority of the solid graphitic material for exposed contact lubrication. Following these preliminary find-

ings a series of long test runs was made with the test machine, and a quantity of data was secured for the purpose of comparing the effectiveness of graphite-grease and solid graphitic material. The test conditions maintained and the observations made were the same as in the tests on graphite-grease, the lubricator shoe pressure being maintained at $3^3/_4$ lb.

Micrometer readings of the thickness of the copper wire were taken at 36 points on the wheel ring and the wear calculated from each measurement. The average wire wear then was determined and recorded graphically.

Results of the laboratory tests are summarized graphically on the chart of Fig. 8, which shows also the wire wear experienced in actual railroad operation under the most favorable conditions of graphite-grease lubrication near locomotive terminals. Curves representing laboratory findings should not be compared directly with data secured on the railroad because many variables, such as the type and flexibility of contact line suspension, weather and frost conditions, etc., enter into actual operation. Comparison of the 3 curves shown in Fig. 8 indicates, however, that there is a vast difference between the lubricating properties of graphite-grease and a solid graphitic lubricant as used on the test machine. These results indicate the possibility of improvement over the best conditions found in railroad operation. Without considering the curves of Fig. 11 as complete and final evidence, it is only logical to conclude that they indicate a distinct superiority of solid graphitic lubricant.

At the conclusion of the tests using solid lubricant the contact surfaces of the wheel ring and collector strip were of a rich bronze color and polished like a well kept commutator. The glossy surface could not be photographed satisfactorily, although the clearness of a reflected image is shown in Fig. 9.

A further test to prove the effectiveness of solid graphitic lubricant was designed to show its ability to repair a badly damaged and burned contact surface. This test was run when the wheel ring was in very poor condition following an extended run with graphite-grease, which had failed to maintain the contact surface in satisfactory condition. The auxiliary lubricator shoe was installed and the machine put back in operation with a rim speed of 45 miles per hour. The application of solid lubricant immediately reduced the mechanical friction of the contact to a low and constant value, the contact sparking was reduced, and the contact surfaces began to take on a polish.

Another test then was started in which the auxiliary lubricator shoe was tied down a small fraction of an inch with a string until it just cleared contact with the moving wire ring as shown in Fig. 10 at point *A*. Pressure on the current collector shoe was not altered appreciably. Lack of lubrication caused the contact surface to roughen at once, and the increased contact friction overloaded the test wheel drive motor until it finally stalled in a little more than 3 min as indicated at point *B*. The drive motor was a shunt machine with its armature circuit protected by a low external resistance as shown in the small circuit inset in the chart of Fig. 10. During this retarded condition the drive motor armature current more than doubled and its driving torque, of course, increased in like proportion tending to keep the wheel in motion. By the time the wheel had stalled, the fierce arcing and flashing caused by lack of smoothness at the contact had left the contact sur-

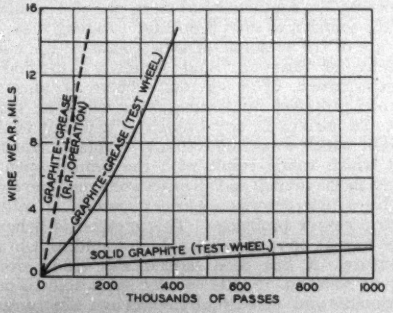


Fig. 8 (above). Comparison of contact line wear with graphite-grease and with solid graphitic lubrication

Fig. 9. Polished contact surface of wheel ring after extensive test with solid lubricant; watch is for reflection only. A similar polish was imparted to the contact shoe

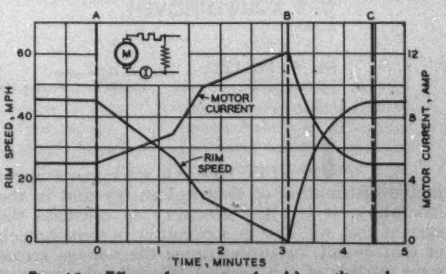


Fig. 10. Effect of removing (at A) auxiliary shoe of solid graphite lubricant while test wheel was rotating, and replacing it (at B) after the resulting increased friction had stalled the wheel

The small diagram shows the connections of the driving motor where I is an ammeter

face in an extremely rough and burned condition, apparently beyond repair.

At point $B$ the string that held the auxiliary lubricator shoe clear of the wheel ring was cut, allowing the auxiliary shoe again to rub against the contact surface of the wheel ring. A small amount of the solid lubricant was rubbed into the surface of the wheel ring by the roll motion of the collector shoe carriage, and the wheel started and accelerated to normal smooth running conditions in 75 sec. The contact surfaces again began to assume a polish, and the solid lubricant on the auxiliary shoe rode smoothly against the wire without being worn after the first few turns of the wheel. This test illustrates the property of a solid lubricant to feed its graphite material automatically onto a rough surface requiring lubrication and to conserve the lubricant when running on a smooth surface. The test illustrates also the very important property of a suitable solid lubricant to repair quickly a badly damaged contact surface. Numerous repetitions of this experiment always developed results similar to those shown in Fig. 10.

### Road Tests of Solid Graphitic Material

Wear on the solid lubricating material used on the auxiliary shoe was practically negligible during the laboratory tests; however, no information was available at the conclusion of these experiments to predict the probable life of such a shoe when used on a locomotive pantograph in main line road service. To complete the study of the solid lubricant several different types of auxiliary shoes were built, installed on the Milwaukee locomotives, and run over the Coast division between Tacoma and Othello, Wash., a distance of 210 miles. The auxiliary lubricator shoes were applied to pantographs of locomotives in both freight and passenger service and gave consistently good results. One auxiliary shoe wore away only $^1/_8$ in. of its solid lubricant during operation under more than 800 miles of trolley wire.

# A New Method of Calculating Circuits

**A "short-circuit current solution" for calculating the currents and voltages in an electrical network has recently been discovered. It is applicable to both d-c and a-c circuits, and reduces the labor of calculations considerably.**

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IN ELECTRICAL work, particularly in the distribution of electrical energy and in the laboratory, it is often necessary to calculate the current that will flow in a circuit or in some branch of that circuit, or to determine the voltage across some portion of the circuit. The straight forward method of solving these problems is to follow the plan originally proposed by Kirchoff,[1] namely, to apply Ohm's law and Kirchoff's laws. This method is sometimes lengthy and involves the solution of a number of simultaneous equations. Other methods involving less mathematical work are also available for the solution of such problems. Among these are the use of the principle of superposition,[2] the cyclic currents of Maxwell,[3] and Kennelly's method of transformation of a circuit.[4] There is also the method proposed by Wenner,[5,6] in which he shows that the current in any branch of a circuit is that current which would flow should a voltage equal to that which would result with the branch open be placed in the branch and all other voltages removed.

This article describes another method of solving electric circuit problems. This method is believed to be new as no reference has been found to it in the literature; it has been named the "short-circuit current solution." It is equally applicable to both continuous and a-c current circuits and also to radio circuits. The method is particularly useful in circuits with 2 or more sources of electromotive force operating in parallel, and the description following applies in such cases. It involves the determination of an entirely fictitious short-circuit current, calculated on the basis of short circuiting the generators or sources of electromotive force through their normal operating impedances. The resulting short-circuit currents are added together vectorally and the total short-circuit current is multiplied by the equivalent parallel impedance of the circuit. This parallel impedance is also fictitious, as the load is normally in series with the source of electromotive force. The result gives the effective voltage across the circuit and it is then a simple matter to determine the current in any branch of the circuit.

As an example, consider first a simple circuit. Assume that there are 2 generators operating in parallel and supplying a load. Let $E_1$ be the voltage and $Z_1$ the internal impedance of one of the genera-

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1. For all references, see numbered list at end of article.

tors, and $E_2$ and $Z_2$, respectively, the voltage and impedance of the second generator. Let $Z_L$ equal the impedance of the load. (The symbols represent complex quantities.) The diagram of connections is shown in Fig. 1. Using Kirchoff's laws, instead of the new method, the solution is as follows:

$$E_1 = I_1Z_1 + I_LZ_L$$
$$E_2 = I_2Z_2 + I_LZ_L$$
$$I_L = I_1 + I_2$$

From these 3 equations are found the currents

$$I_L = \frac{E_1Z_2 + E_2Z_1}{Z_1Z_2 + Z_1Z_L + Z_2Z_L}$$
$$I_1 = \frac{E_1Z_2 + E_1Z_L - E_2Z_L}{Z_1Z_2 + Z_1Z_L + Z_2Z_L}$$
$$I_2 = \frac{E_2Z_1 + E_2Z_L - E_1Z_L}{Z_1Z_2 + Z_1Z_L + Z_2Z_L}$$

and the voltage drop $E$ across the circuit

$$E = I_LZ_L = \frac{E_1Z_2Z_L + E_2Z_1Z_L}{Z_1Z_2 + Z_1Z_L + Z_2Z_L}$$

Now let us solve this circuit, Fig. 1, by the short-circuit current solution. Consider each generator separately and assume that it is short circuited through its own internal impedance as in Fig. 2*a*. Then the corresponding fictitious short-circuit currents that will flow are:

$$I_{1s} = \frac{E_1}{Z_1} \quad \text{and} \quad I_{2s} = \frac{E_2}{Z_2}$$

The equivalent voltage $E$ across the circuit under these conditions is given by multiplying the sum of these 2 currents by the parallel impedance of the circuit. This fictitious impedance includes the internal impedances of the 2 generators and of the load all assumed in parallel as stated before, i. e., the impedance between points $A$ and $B$ as shown in Fig. 2*b*.

$$E = (I_{1s} + I_{2s})\frac{Z_1Z_2Z_L}{Z_1Z_2 + Z_1Z_L + Z_2Z_L}$$

Substituting the values of $I_{1s}$ and $I_{2s}$ and clearing the voltage across the circuit is found to be:

$$E = \frac{E_1Z_2Z_L + E_2Z_1Z_2}{Z_1Z_2 + Z_1Z_L + Z_2Z_L}$$

The load current $I_L$ is given by

$$I_L = \frac{E}{Z_L} = \frac{E_1Z_2 + E_2Z_1}{Z_1Z_2 + Z_1Z_L + Z_2Z_L}$$

The generator currents $I_1$ and $I_2$ are

$$I_1 = \frac{E_1 - E}{Z_1} = \frac{E_1Z_2 + E_1Z_L - E_2Z_L}{Z_1Z_2 + Z_1Z_L + Z_2Z_L}$$
$$I_2 = \frac{E_2 - E}{Z_2} = \frac{E_2Z_1 + E_2Z_L - E_1Z_L}{Z_1Z_2 + Z_1Z_L + Z_2Z_L}$$

## Derivation of General Equations

It is evident from the above example that the use of the short-circuit current method gives results that are identical with those obtained by the classical methods. The relations used in the short-circuit current method may be derived directly from Kirchoff's laws as follows:

In order to make this derivation general assume that instead of the 2 generators shown in Fig. 1, there are $n$ generators operating in parallel, each with its own internal impedance and that the equivalent load impedance is represented by $Z_L$ as in Fig. 1. Then by Kirchoff's laws the equations for the currents and voltages are

$$I_1 + I_2 + I_3 \ldots + I_n = I_L \tag{1}$$
$$E_1 - I_1Z_1 = E_2 - I_2Z_2 = E_3 - I_3Z_3 = \ldots = E_n - I_nZ_n = I_LZ_L = E \tag{2}$$

Substitute into the electromotive-force relation of generator 1 the value of $I_1$ from eq 1 and divide by $Z_1$ the generator impedance

$$\frac{E_1}{Z_1} + I_2 + I_3 + \ldots + I_n - I_L = \frac{E_1}{Z_1} \tag{3}$$

The following relations for the other $n - 1$ generators are obtained directly from eq 2

$$\frac{E_2}{Z_2} - I_2 = \frac{E}{Z_2} \tag{4}$$
$$\frac{E_3}{Z_3} - I_3 = \frac{E}{Z_3} \tag{5}$$
$$\ldots\ldots\ldots$$
$$\ldots\ldots\ldots$$
$$\frac{E_n}{Z_n} - I_n = \frac{E}{Z_n} \tag{6}$$

For the load there results

$$+ I_L = \frac{E}{Z_L} \tag{7}$$

The algebraic sum of eq 3 to 7 inclusive gives

$$\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3} + \ldots + \frac{E_n}{Z_n} = E\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots + \frac{1}{Z_n} + \frac{1}{Z_L}\right) \tag{8}$$

But $\frac{E_1}{Z_1}$; $\frac{E_2}{Z_2}$ etc., have been defined as the fictitious short-circuit currents; $I_{1s}$; $I_{2s}$; etc.; and the sum of the reciprocals of the impedances multiplying $E$ is $\frac{1}{Z_p}$ where $Z_p$ equals the equivalent parallel impedance. It follows that

$$(I_{1s} + I_{2s} + I_{3s} + \ldots + I_{ns})Z_p = E \tag{9}$$

Furthermore, if the equations of Kirchoff's laws for the circuit are written in complex notation, then solved and compared with the derived equations for

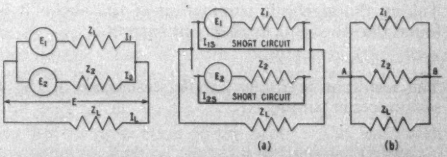


**Fig. 1. Diagram of connections for a simple circuit**

**Fig. 2. Diagram of connections of simple circuit illustrating method of obtaining (a) fictitious short-circuit currents, and (b) fictitious impedance**

the fictitions short-circuit current solution, the 2 results will be found identical term for term.

Therefore it is clear that the short-circuit current solution is based directly upon the fundamental laws of the electrical circuit, and is theoretically correct.

### Numerical Example—D-C Problem

In order to illustrate the simplicity of the short-circuit current solution 2 numerical examples are given below. Assume that in the circuit of Fig. 3

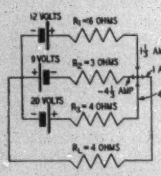


Fig. 3. Circuit of the example which involves 3 d-c generators or batteries

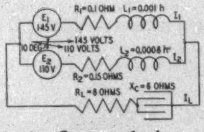


Fig. 4. Circuit of the example which involves 2 a-c generators

there are 3 d-c generators or batteries with their respective internal resistances supplying the load $R_L$ connected as shown. Determining the currents and voltage drops in this circuit in accordance with the short-circuit current solution the 3 fictitious short-circuit currents are first calculated.

$$I_{1s} = \frac{E_1}{R_1} = \frac{12}{6} = +2 \text{ amp}$$

$$I_{2s} = \frac{E_2}{R_2} = -\frac{9}{3} = -3 \text{ amp}$$

$$I_{3s} = \frac{E_3}{R_3} = \frac{20}{4} = +5 \text{ amp}$$

Taking the algebraic summation of the above it is found that the total short-circuit current is

$$I_{1s} + I_{2s} + I_{3s} = +4 \text{ amp}$$

The resultant fictitious parallel resistance of the 4 resistances in parallel is

$$R_p = 1 \text{ ohm}$$

The voltage drop $E$ across the circuit is

$$E = (I_{1s} + I_{2s} + I_{3s})R_p = 4 \text{ volts}$$

and the currents are

$$I_L = \frac{E}{R_4} = \frac{4}{4} = 1 \text{ amp}$$

$$I_1 = \frac{E_1 - E}{R_1} = \frac{12 - 4}{6} = 1^1/_3 \text{ amp}$$

$$I_2 = \frac{E_2 - E}{R_2} = \frac{-9 - 4}{3} = -4^1/_3 \text{ amp}$$

$$I_3 = \frac{E_3 - E}{R_3} = \frac{20 - 4}{4} = +4 \text{ amp}$$

These results check those obtained by Kirchoff's laws exactly and it is clearly evident that the amount of calculation involved in the use of the short-circuit current solution is considerably less than that using any of the standard methods.

### Numerical Example—A-C Problem

As another example of the use of the short-circuit current solution for calculating circuits the 60 cycle circuit shown in Fig. 4 has been chosen; in this case it is desired to know the currents carried by the load and the generators. Two generators, $E_1$ and $E_2$, respectively, supply the load. The electromotive force of generator $E_1$ is 145 volts and its internal resistance is 0.1 ohm and inductance 0.001 h, making its reactance $X_1 = 0.3768$ ohms and its impedance $Z_1 = 0.38984$ ohm. Generator $E_2$ has an electromotive force of 110 volts and its internal resistance is 0.15 ohms and its inductance 0.0008 h, giving $X_2 = 0.30144$ ohm and $Z_2 = 0.3367$ ohm. These 2 generators are operating in parallel but for the purpose of the problem it will be assumed that they are not in phase and that voltage $E_2$ lags $E_1$ by 10 deg. The load consists of a resistance and capacitor in series, the values of which are $R_L = 8$ ohms and $X_C = 6$ ohms, respectively, and its impedance is $Z_L = 10$ ohms.

The load current was calculated by Kirchoff's laws and also by the short-circuit current method. A calculating machine was used in carrying out the computations in order to insure accuracy. The steps and the results obtained using the fictitious short-circuit current solution are given below.

$$I_{1s} = \frac{E_1}{Z_1} = 371.9446 \text{ amp lagging}$$

$$I_{2s} = \frac{E_2}{Z_2} = 326.70022 \text{ amp lagging}$$

Resolving these into their real and imaginary components and remembering that $E_2$ lags $E_1$, the base vector, by 10 deg, the following currents are obtained:

| | Real Component | Imaginary Component |
|---|---|---|
| $I_{1s}$ | 95.4088 | −359.5004 |
| $I_{2s}$ | 92.5446 | −313.3192 |
| $I_{1s} + I_{2s}$ | 187.9534 | −672.8196 |

$$I_{1s} + I_{2s} = 698.579 \text{ amp}$$

The fictitious admittance of the parallel circuit consisting of $Z_1$, $Z_2$, and $Z_L$ is

$$Y = 2.0611 - j\,5.0783 = 5.4806 \text{ mhos}$$

and

$$Z_p = 0.06862 + j\,0.16907 \text{ ohm}$$

The resultant voltage across the circuit is therefore

$$E = (I_{1s} + I_{2s})Z_p = 126.6478 - j\,14.3917 = 127.463 \text{ volts}$$

Thus the phase relation of $E$ with respect to $E_1$ and $E_2$ is readily determined. The currents $I_1$ and $I_2$ and their phase relations as well as that of $I_L$ are as follows:

$$I_1 = \frac{E_1 - E}{Z_1} = \frac{(145 + j\,0) - (126.6478 - j\,14.3917)}{0.1 + j\,0.3768}$$

$$= 47.7569 - j\,36.0311 = 59.8244 \text{ amp}$$

Since $E_1$ is taken as the base vector and $E_2$ lags $E_1$ by 10 deg

$E_3 = 110 = 108.3289 - j\,19.1013$ volts

Then

$$I_1 = \frac{E_3 - E}{Z_1} = \frac{(108.3289 - j\,19.101) - (126.6478 - j\,14.3917)}{0.15 + j\,0.30144}$$
$$= -36.7616 + j\,42.4785 = 56.1769 \text{ amp}$$

The load current is the vector sum of the generator currents and may be calculated directly

$I_L = I_1 + I_3$

$I_L = 10.9953 + j\,6.4475 = 12.7463$ amp

Or the load current may be determined from the relation

$$I_L = \frac{E}{Z_L} = \frac{126.7836 - j\,14.3917}{8 - j\,6} = 12.7463 \text{ amp}$$

Using the method of Kirchoff's laws exactly the same value for the load current is obtained.

Many other examples have been worked out and in every instance the short-circuit current solution has given the correct results. It is felt, however, that the above examples clearly illustrate the application of the method to circuit problems.

As stated above this new method is equally applicable to both continuous and a-c circuit problems. It may also be applied to polyphase circuits where the load is balanced, and to high frequency circuits. The method reduces the labor of circuit calculation considerably and should form a useful tool in the hands of the engineer and physicist.

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# Radio Direction Finder for Use on Airplanes

THE navigation of airplanes in fog or under conditions of low visibility is becoming more and more dependent upon the use of directional radio devices. As used in the United States, directional radio equipment takes 2 forms: (*a*) the directive range beacon used on the established airways, requiring no directional radio equipment on the airplanes using the service, and (*b*) the direction finder, used on airplanes engaged in cross-country flying independent of the airways. The use of the direction finder on airplanes has not been extensive. There has, however, been increasing demand for a direction finder designed specifically for airplane use. The U.S. Bureau of Standards, in its capacity as the research division of the aeronautics branch of the department of commerce, has developed a simple direction finder for this purpose. The following description is quoted in full from U.S. Bureau of Standards *Technical News Bulletin* for August 1933, p. 83–4.

Recent flight tests made by the bureau have shown the complete practicability of the device. The direction finder uses visual rather than aural indication, and has no moving parts except the rotatable loop antenna. Positive visual indication right and left of course is given, as well as direction sense (i. e., freedom from 180 deg ambiguity), without special manipulation by the pilot. The characteristics of the incoming signals are not destroyed, thus allowing simultaneous utilization of the incoming signals for message reception as well as for direction determination.

This direction finder operates on any received station; and uses any receiving set, which may be the regular receiving set already carried by the airplane. It is only necessary to supply a small loop antenna and a compact converter unit. This converter unit furnishes a current of such a nature that an instrument shows a zero-center deflection when the heading of the airplane coincides with the direction of the station being received, and deflects to the right or left according to the departure of the airplane heading from this direction.

An interesting feature of the direction finder is its operation under conditions of bad atmospherics ("static") and ignition interference. When on-course signals are received there is little effect, and when off-course signals are received, any change in the indication tends toward an on-course indication. No violent kicking of the course indicator occurs.

The model recently tested weighs about 10 lb, is about 6x8x10 in. in size; both weight and size can be considerably reduced through mechanical refinement.

The direction finder depends for its operation upon the production of 2 modified figure-of-eight space patterns from one loop antenna, switched on alternately. For a given position of the loop antenna with respect to an in-coming radio wave a zero-center course indicator is made to deflect to the right in proportion to one field pattern and to the left in proportion to the other. The switching frequency is high so that the net pointer deflection is the difference of these 2 and is at zero center when the 2 are equal, swinging right or left depending upon whether the first or second deflection is the greater. (This is determined by the direction of the incoming wave.)

The possibility of error from a change in the component parts of the direction finder is remote, since the converter unit does not depend for accuracy upon the amplification of a vacuum tube, or the balance of the amplification of 2 tubes. A failure in the radio receiving set cannot introduce a course error. There are no mechanical rotating parts, which makes for lower cost and increased dependability.

The most advantageous feature of this direction

finder is its almost completely automatic operation. The loop antenna does not require any careful tuning or phasing operations, since it constitutes the sole means of pick-up for the reception of transmissions.

The direction finder was installed and tested on an airplane at the College Park, Md., experimental field. Test flights were made between Washington and Baltimore, observing on the broadcasting stations of those 2 cities. In these test flights the direction finder was used as a homing device. The operation of the direction finder was entirely satisfactory, indications right and left of the heading of the aircraft being very steady and definite. On passing over the stations toward which the flights were made, positive localizing action was given by the reversal of the action of the course indicator. The pilot experienced no difficulty either in accurately following the indications of course, or in locating the transmitting stations, although the locations of the station toward which the flights were made were unknown to him. At normal volumes, a 10 deg variation of heading, right or left, produced full scale readings right or left on the course indicator; this variation can be readily increased or decreased as desired.

This direction finder was designed primarily for use on airplanes, where dependability, ease of operation, sensitivity, and compactness are of primary importance, but it is also adaptable to marine use. It may be used as a homing device or for position finding by means of cross bearings.

# Power Limits of Synchronous Machines

**This paper presents methods for determining steady state power limits of electric power systems composed of shunt impedance loads and any number of ideal cylindrical-rotor synchronous machines. In order to apply these methods to practical systems, the actual machines are represented by equivalent ideal machines corresponding to a given operating condition. The methods are said to be simpler and easier to apply than previous methods.**

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METHODS for determining steady state power limits of systems composed of shunt impedance loads and ideal cylindrical-rotor synchronous machines are presented in this paper. These methods, based upon a stability criterion that is independent of machine inertias, are simpler and easier to apply than any previously published. They are applicable to systems of any size. It seems appropriate, therefore, to submit them at this time, when the trend in system operation and design is toward a narrower margin between transient and steady state power limits.

Stability limits of systems are not determined directly; but, for any assumed conditions of loading, the systems are tested for stability and found to be stable or unstable. The incremental power equations with which the tests are made are developed for systems consisting of 2, 3, 4, and 5 machines and can be extended to those having any number of machines. These equations are arranged so that it is necessary to determine only the signs of the incremental power coefficients for the various machines, to decide whether a system is stable or unstable.

In practical systems, synchronous machines, whether of the cylindrical-rotor or salient-pole type, can be reduced to equivalent or near equivalent ideal machines corresponding to given operating conditions. Similarly, loads may be represented approximately by shunt impedances alone or in combination with synchronous machines. Some principles that can be used in combining machines or machines and impedance shunts are stated in Appendix III; an equivalent circuit for the synchronous condenser, saturation included, is given in Appendix IV; and in Appendix V is presented an equivalent circuit for a constant power induction machine for use when testing the synchronous machines of the system for stability. In a companion paper[1] by S. B. Crary, the methods presented here are generalized and applied to induction machines, composite loads, and interconnected systems.

There are at least 2 conditions under which the steady state stability limits of a system, or part of a system, are of importance in determining loads that normally can be transmitted. When total loss of power can be tolerated, steady state stability limits are of primary importance in determining the maximum power that can be delivered to a load center. When firm power is required, steady state stability limits indicate the power limits that may be ap-

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1. For all numbered references see bibliography at end of paper.

proached by improved apparatus and system arrangements.

This last condition is of somewhat recent origin. In the past, where 2 or more lines were operated in parallel between important load centers, it usually was found that the steady state stability limit was well above the limit of the load that could be carried through a severe disturbance in which a line was switched out. However, with faster and better switching and relaying, and more advantageous arrangements of equipment and circuits, transient stability limits can be increased to such an extent that they frequently approach the maximum power that can be transmitted under steady state conditions after the disturbance. Thus, the steady state stability limit is often of importance as the upper limit of power that can be transmitted through a disturbance, and consequently is helpful in determining the safe operating load.

## Cut-and-Try Method

The method proposed for determining the steady state stability limits of a system of interconnected synchronous machines and shunt loads is a "cut-and-try" one. The stability limit of the system, of a certain machine, or of a part of the system is not determined directly, but for any assumed condition of loading the system is tested for stability and pronounced either stable or unstable. If the system is found to be stable under the conditions assumed, the load is increased and the test repeated. The stability limit will lie between the greatest load for which the system is stable and the smallest for which it is unstable.

The steady state power limit calculated in this way is not a practical operating limit, since any disturbance, even a small one such as would be occasioned by a switching operation or load change, might cause instability. Consequently, systems must operate with loadings less than those corresponding to the calculated limits. Also, it is evident that there is a different power limit for each set of operating conditions on a system. Frequently a different distribution of power among the generators will raise the stability limit appreciably.

The method is applicable to a system in which there are shunt impedance loads and any number of cylindrical-rotor synchronous machines having no saturation. It may be applied also to a system in which the following assumptions can be made:

1. Synchronous machines can be treated as cylindrical-rotor machines with reactances that remain constant during the test for stability.

2. Loads can be represented by shunt impedances, equivalent synchronous motors, or synchronous motors combined with impedance networks.

3. Groups of synchronous machines can be combined into one equivalent machine. (Appendix III.)

4. Portions of the system can be replaced by a single equivalent machine. (Appendix III.)

## Criterion for Stability

The criterion for steady state stability used in this paper is that an increase in the electric power of any machine accompany an increase in the angular displacement of its rotor in the direction of rotation for a generator and opposite to the direction of rotation for a motor, field currents of all machines of the system remaining constant and the governor settings being readjusted to maintain system frequency.

An increment of mechanical load added to the shaft of a machine, say a motor, will cause a momentary slowing down of the motor thereby increasing the relative angular displacement of its rotor. If the electrical input to the motor be increased by this increase in angle, the motor will cease to slow down, and if the governors of one or more of the other machines have been readjusted to supply the additional power required, so that frequency will be maintained, the motor will be stable. If the electrical output of the generator whose governor has been readjusted to supply the additional power also is increased with an increase in angle between generator and motor, the generator likewise will be stable.

A system will be stable if all the machines of the system are stable; it will be unstable if any machine is unstable.

## Test for Stability

The electric power of a synchronous machine may be expressed in terms of its internal voltage, the internal voltages of the other machines of the system, the angles between these voltages, and the impedances of the system. The internal voltage of a machine for steady state calculations is the voltage behind synchronous reactance, or some equivalent reactance to be discussed later. In a cylindrical-rotor machine of zero saturation, the internal voltage is in quadrature with the axes of the poles. Angular displacement between the internal voltages of 2 such synchronous machines is, therefore, the same as that between their corresponding rotor positions.

Equations for determining the stability of a machine take their simplest form when it is assumed that the increment of power supplied to a motor is furnished by one generator only, or the increment of power furnished by a generator is taken by one motor only. Division of load in accordance with governor characteristics is discussed in a companion paper.[1] The assumption that the increment of power is supplied by one machine only is usually acceptable, since in general it will result in slightly lower steady state stability limits than would be obtained if the load were proportioned in accordance with the load-speed characteristics of the machines.

Therefore, to test for stability it is assumed that all the machines except 2, one to be treated as a generator and the other as a motor, are fixed power machines and consequently will take up angular positions to maintain constant power as the angle between the generator and the motor is increased. If the electric power of either generator or motor decreases with an increase in the angle between their internal voltages, the system is unstable. If the electric power of both generator and motor increases with an increase in angle, both machines are stable for the condition imposed. As this test applied to

only 2 machines at a time, theoretically it is necessary to repeat it, taking the machines 2 at a time so that each machine is tested with every other machine. Practically, however, one test is usually sufficient, that for the 2 machines between which there is the largest angular displacement as determined from the system voltage vector diagram. This test for stability is equivalent to increasing the mechanical load on the motor and the mechanical input to the generator by slight amounts. The procedure of increasing the angular displacement between motor and generator that would result from an increase in mechanical load is the means selected for testing, since it can be applied directly by means of mathematical equations for power flow.

Wagner and Evans have shown[2] that it is theoretically possible under certain conditions to operate a machine when its electric power decreases with an increase in its relative angular displacement. Such operation is possible if the disturbance be caused by an increase in angular displacement, the inertias of the machines being such that the initial operating conditions can be regained. However, should the increase in angular displacement be caused by an increment of load, it would not be possible for the system to return to a state of equilibrium at normal system frequency by readjustment of governor settings.

## Representation of Machines and Loads

It is customary in the one-line diagram of a system to represent a synchronous machine by a voltage to neutral acting through a series impedance. For a cylindrical-rotor machine of negligible saturation, this representation will give accurate results. The salient-pole machine with and without saturation, and the cylindrical-rotor machine with saturation, cannot be represented by a single value of impedance for all conditions of steady state operation. Although no one value of impedance can be used to represent a synchronous machine under all conditions of operation, corresponding to each condition there will be a particular value of equivalent impedance that will give the same response (change in power factor and current) for a small change in terminal voltage as the actual machine connected to the system. This subject is treated in another paper.[3]

In the present paper it will be assumed that there is a value of equivalent impedance for each synchronous machine corresponding to any given operating condition, which will remain constant during the test for steady state stability. An alternate method of determining the equivalent reactance to be used for a condenser operating at specified terminal voltage and field current is given in Appendix IV.

Lighting loads, loads due to magnetizing currents in transformers, and line charging currents can be represented by shunt impedances. An induction machine, when treated as a fixed power machine, can be represented by an impedance network in combination with a synchronous machine. An equivalent circuit for the induction machine as a fixed power machine is given in Appendix V.

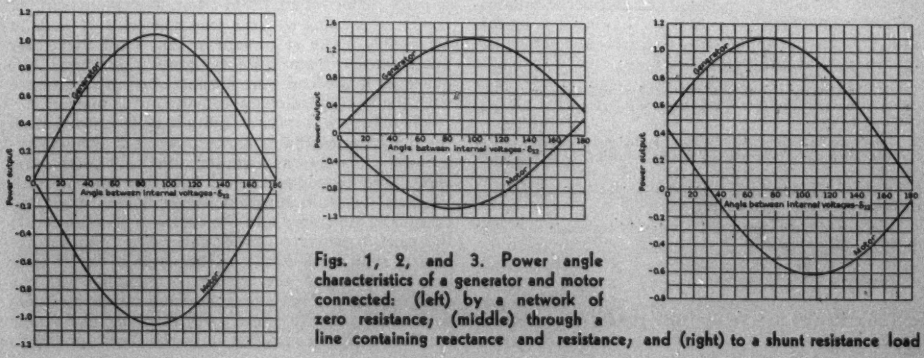
Methods given in this paper are extended in a companion paper[1] to the treatment of induction machines and the effects of composite loads upon the stability of the other machines of the system. The composite load may be composed of induction machines, synchronous machines, and impedance shunts; or it may represent an interconnected system or a portion of the system. The induction machine is treated both as a fixed and as a variable shaft-load machine, and may itself be tested for asynchronous stability.

## Power Equations for Synchronous Machines

*Two Synchronous Machines.* When a system has only 2 synchronous machines, one a motor and the other a generator, the equations for their power outputs in terms of the scalar values of the internal voltages and driving point and transfer impedances are:[4]

$$P_1 = \frac{E_1^2}{Z_{11}} \sin \alpha_{11} + \frac{E_1 E_2}{Z_{12}} \sin (\delta_{12} - \alpha_{12}) \qquad (1)$$

$$P_2 = \frac{E_2^2}{Z_{22}} \sin \alpha_{22} + \frac{E_1 E_2}{Z_{12}} \sin (\delta_{21} - \alpha_{12}) \qquad (2)$$




Figs. 1, 2, and 3. Power angle characteristics of a generator and motor connected: (left) by a network of zero resistance; (middle) through a line containing reactance and resistance; and (right) to a shunt resistance load

where

$P_1$ = power out of machine 1
$E_1$ = magnitude of voltage behind synchronous reactance of machine 1
$Z_{11}$ = magnitude of driving point impedance of machine 1, i. e., ratio of voltage at 1 to current at 1 with no other voltage applied to the system
$Z_{12} = Z_{21}$ = magnitude of transfer impedance between machines 2 and 1, i. e., ratio of voltage at 1 to current at 2, with no other voltage applied to the system
$\theta_{12} = \theta_{21}$ = impedance angle of $Z_{21}$
$\alpha_{12} = \alpha_{21} = 90 - \theta_{21}$
$\delta_1$ = angle $E_1$ makes with the reference vector
$\delta_{12} = \delta_1 - \delta_2$

Positive power is taken as power out of a machine. Normal power for a generator therefore will be positive, and for a motor negative.

Replacing $\delta_{21}$ in eq 2 by $-\delta_{12}$,

$$P_2 = \frac{E_2^2}{Z_{22}} \sin \alpha_{22} - \frac{E_1 E_2}{Z_{12}} \sin (\delta_{12} + \alpha_{12}) \qquad (2a)$$

It can be seen by inspection of eq 1 that the generator power $P_1$ will be a maximum when $\delta_{12} = 90 + \alpha_{12}$; and from eq 2*a* that $P_2$, the motor power, will reach a maximum when $\delta_{12} = 90 - \alpha_{12}$.

When there is no resistance in the network between $E_1$ and $E_2$, $\theta_{12} = 90°$ and $\alpha_{12} = 0$. The power outputs of both generator and motor will be maximum when $\delta_{12} = 90°$; and since $\alpha_{11} = \alpha_{22} = 0$, the first term on the right-hand side in eqs 1 and 2*a* will disappear, and

$$P_{1(\max.)} = \frac{E_1 E_2}{X_{12}}$$

$$P_{2(\max.)} = -\frac{E_1 E_2}{X_{12}}$$

The power-angle characteristics of a generator and motor with constant internal voltages, connected by a network having no resistance, are given in Fig. 1. For this case the power output of the motor is the negative of the power output of the generator, and both are maximum when $\delta_{12}$ is 90°.

When the only resistance is series resistance, the impedance angle of the transfer impedance between generator and motor, $\theta_{12}$, will be less than 90° and $\alpha_{12}$ will be positive. The power output of the generator will be maximum when $\delta_{12}$ is greater than 90°, and of the motor when $\delta_{12}$ is less than 90°; i. e., for the generator when $\delta_{12} = 90 + \alpha_{12} = 180 - \theta_{12}$, and for the motor when $\delta_{12} = 90 - \alpha_{12} = \theta_{12}$.

Figure 2 gives the power-angle characteristics of the generator and motor with constant internal voltages, connected by series impedance, i. e., resistance and reactance. The power output of the generator will be maximum when $\delta_{12}$ is greater than 90°, and of the motor when $\delta_{12}$ is less than 90°.

When the only resistance is shunt resistance, $\theta_{12}$ will be greater than 90° and $\alpha_{12}$ negative. In this case, the power output of the motor will be maximum when $\delta_{12}$ is greater than 90°, and of the generator when $\delta_{12}$ is less than 90°.

Figure 3 gives the power-angle characteristics of the generator and motor with constant internal voltages, connected to a shunt impedance load, but with no resistance in series. Under these conditions, the power output of the generator will be maximum when $\delta_{12}$ is less than 90°, and of the motor when $\delta_{12}$ is greater than 90°.

In a system having resistance, the particular machine that will reach its maximum power first, as $\delta_{12}$ is increased, will depend upon whether the resistance is in series or shunt. When resistance is both in series and shunt, $\alpha_{12}$ may be positive, negative, or zero.

*Three Synchronous Machines.* Although the steady state stability limit of the 2-machine system can be found readily from the general power equations, such is not the case where more than 2 machines are operating under specified conditions. When there are 3 machines the power equations become:

$$P_1 = \frac{E_1^2}{Z_{11}} \sin \alpha_{11} + \frac{E_1 E_2}{Z_{12}} \sin (\delta_{12} - \alpha_{12}) + \frac{E_1 E_3}{Z_{13}} \sin (\delta_{13} - \alpha_{13}) \qquad (3)$$

$$P_2 = \frac{E_2^2}{Z_{22}} \sin \alpha_{22} + \frac{E_1 E_2}{Z_{12}} \sin (\delta_{2} - \alpha_{12}) + \frac{E_2 E_3}{Z_{23}} \sin (\delta_{23} - \alpha_{23}) \qquad (4)$$

$$P_3 = \frac{E_3^2}{Z_{33}} \sin \alpha_{33} + \frac{E_1 E_3}{Z_{13}} \sin (\delta_{31} - \alpha_{13}) + \frac{E_2 E_3}{Z_{23}} \sin (\delta_{32} - \alpha_{23}) \qquad (5)$$

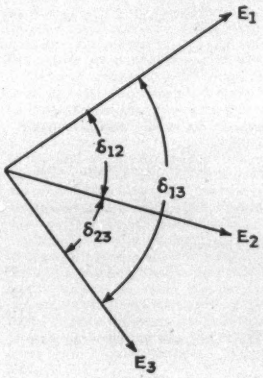



Fig. 4. Voltage vector diagram

These expressions are the equations for power output from machines 1, 2, and 3. Figure 4 gives the voltage vector diagram of the internal voltages of the 3 machines and their displacement angles for an assumed operating condition which may or may not be stable. In the illustration used, machine 1 leads machines 2 and 3, and machine 2 leads machine 3, the greatest phase displacement being between generator 1 and motor 3.

The test for stability will be applied by arbitrarily increasing the angle $\delta_{13}$ between machines 1 and 3 by an increment, $\Delta\delta_{13}$. During the test the power output (or input) of machine 2 will remain constant. When the angle $\delta_{13}$ is increased, machine 2 to maintain constant power must take up a new angular position relative to machines 1 and 3. If the power out of the generator 1 and the power into the motor 3 both increase with an increase in $\delta_{13}$, the power out of machine 2 remaining constant, machines 1 and 3 are operating at an angle, $\delta_{13}$, less than their maximum power angle. If the power for either machine decreases with an increase in $\delta_{13}$, the machines are operating at an angle greater than their maximum power angle and the system is unstable.

## Power Increment Equations

Equations for the increments of power for small changes in machine angles are obtained from the power output eqs 3, 4, and 5. For small changes in the relative angular positions of the internal voltages of the 3 machines, the power equations may be differentiated and the differentials replaced by infinitesimals giving,

$$\Delta P_1 = \frac{E_1E_2}{Z_{12}}\cos(\delta_{12}-\alpha_{12})\,\Delta\delta_{12} + \frac{E_1E_3}{Z_{13}}\cos(\delta_{13}-\alpha_{13})\,\Delta\delta_{13} \quad (6)$$

$$\Delta P_2 = \frac{E_1E_2}{Z_{12}}\cos(\delta_{21}-\alpha_{12})\,\Delta\delta_{21} + \frac{E_2E_3}{Z_{23}}\cos(\delta_{23}-\alpha_{23})\,\Delta\delta_{23} \quad (7)$$

$$\Delta P_3 = \frac{E_1E_3}{Z_{13}}\cos(\delta_{31}-\alpha_{13})\,\Delta\delta_{31} + \frac{E_2E_3}{Z_{23}}\cos(\delta_{32}-\alpha_{23})\,\Delta\delta_{32} \quad (8)$$

Let

$$K_{12} = \frac{E_1E_2}{Z_{12}}\cos(\delta_{12}-\alpha_{12}) \qquad K_{22} = \frac{E_2E_3}{Z_{23}}\cos(\delta_{23}-\alpha_{23})$$

$$K_{13} = \frac{E_1E_3}{Z_{13}}\cos(\delta_{13}-\alpha_{13}) \qquad K_{21} = \frac{E_1E_2}{Z_{12}}\cos(\delta_{21}-\alpha_{12})$$

$$K_{21} = \frac{E_1E_2}{Z_{12}}\cos(\delta_{21}-\alpha_{12}) \qquad K_{23} = \frac{E_2E_3}{Z_{23}}\cos(\delta_{23}-\alpha_{23})$$

The equations for power change become

$$\Delta P_1 = K_{12}\Delta\delta_{12} + K_{13}\Delta\delta_{13} \quad (9)$$

$$\Delta P_2 = K_{21}\Delta\delta_{21} + K_{22}\Delta\delta_{23} \quad (10)$$

$$\Delta P_3 = K_{31}\Delta\delta_{31} + K_{32}\Delta\delta_{32} \quad (11)$$

Replacing $\Delta\delta_{21}$ by $-\Delta\delta_{12}$, $\Delta\delta_{31}$ by $-\Delta\delta_{13}$, and $\Delta\delta_{23}$ by $-\Delta\delta_{32}$ gives

$$\Delta P_1 = K_{12}\Delta\delta_{12} + K_{13}\Delta\delta_{13} \quad (12)$$

$$\Delta P_2 = -K_{21}\Delta\delta_{12} + K_{22}\Delta\delta_{32} \quad (13)$$

$$\Delta P_3 = -K_{31}\Delta\delta_{13} - K_{32}\Delta\delta_{32} \quad (14)$$

Since $P_2$ remains constant during the test for stability, $\Delta P_2 = 0$. Making this substitution in eq 13,

$$-K_{21}\Delta\delta_{12} + K_{22}\Delta\delta_{23} = 0 \quad (15)$$

$$\therefore\ \Delta\delta_{12} = \frac{K_{22}}{K_{21}}\,\Delta\delta_{32} \quad (16)$$

From Fig. 4

$$\Delta\delta_{12} + \Delta\delta_{32} = \Delta\delta_{13} \quad (17)$$

Solving eqs 16 and 17 for $\Delta\delta_{12}$ and $\Delta\delta_{32}$ in terms of $\Delta\delta_{13}$,

$$\Delta\delta_{13} = \frac{K_{22}}{K_{22}+K_{21}}\,\Delta\delta_{13} \quad (18)$$

$$\Delta\delta_{32} = \frac{K_{21}}{K_{22}+K_{21}}\,\Delta\delta_{13} \quad (19)$$

Substituting these values of $\Delta\delta_{12}$ and $\Delta\delta_{32}$ in the equations for $\Delta P_1$ and $\Delta P_3$, 12 and 14, give

$$\Delta P_1 = \left[\frac{K_{12}K_{22}+K_{13}K_{22}+K_{13}K_{21}}{K_{22}+K_{21}}\right]\Delta\delta_{12} \quad (20)$$

$$\Delta P_3 = -\left[\frac{K_{31}K_{22}+K_{31}K_{21}+K_{32}K_{21}}{K_{22}+K_{21}}\right]\Delta\delta_{12} \quad (21)$$

If the signs of the expressions in brackets in eqs 20 and 21 are both positive, machines 1 and 3 are operating at an angle less than their maximum power angle; if the sign of either expression is negative, the machines are operating beyond their maximum power angle and the system is unstable.

The same equations may be used to determine the stability of machine 2, since the assignment of numbers to the machines was arbitrary. Interchanging subscripts 2 and 3 in eqs 20 and 21, the equations become those for machines 1 and 2. Thus,

$$\Delta P_1 = \left[\frac{K_{12}K_{22}+K_{12}K_{23}+K_{13}K_{21}}{K_{23}+K_{21}}\right]\Delta\delta_{12} \quad (20a)$$

$$\Delta P_2 = -\left[\frac{K_{21}K_{22}+K_{21}K_{21}+K_{23}K_{21}}{K_{22}+K_{21}}\right]\Delta\delta_{12} \quad (21a)$$

The values of the $K$'s, however, used in eqs 20a and 21a should correspond to the original subscripts.

Relations similar to those of eqs 20 and 21 are given in Appendix I for 4 machines and in Appendix II for 5 machines.

## Procedure for Determining Power Limit

1. Reduce the system to its simplest form, preserving the identity of points at which specified voltages are maintained. The reduction of a system to its simplest form and some of the principles used in combining machines, or machines and shunts, are given in Appendix III.

2. For an assumed load, under given operating conditions, calculate the internal voltages of all machines in magnitude and phase.

3. Calculate the transfer impedances between all machines.

4. From the magnitudes of the internal voltages, the $\delta$-angles between them, and the magnitudes and $\alpha$-angles of the transfer impedances, calculate the $K$'s.

5. Substitute the $K$'s in the formulas to test for stability.

6. If the system is stable for the load assumed, increase the load and proceed as before until the stability limit is reached.

## Appendix I—4 Machines

For 4 machines the power increment equations, for a test of machines 1 and 4, are:

$$\Delta P_1 = K_{12}\Delta\delta_{12} + K_{13}\Delta\delta_{13} + K_{14}\Delta\delta_{14}$$
$$\Delta P_2 = K_{21}\Delta\delta_{21} + K_{22}\Delta\delta_{23} + K_{24}\Delta\delta_{24}$$
$$\Delta P_3 = K_{31}\Delta\delta_{31} + K_{32}\Delta\delta_{32} + K_{34}\Delta\delta_{34}$$
$$\Delta P_4 = K_{41}\Delta\delta_{41} + K_{42}\Delta\delta_{42} + K_{43}\Delta\delta_{43}$$

Replacing $\Delta P_2$ and $\Delta P_3$ by 0, $\Delta\delta_{21}$ by $-\Delta\delta_{12}$, $\Delta\delta_{31}$ by $-\Delta\delta_{13}$, etc.,

$$\Delta P_1 = K_{12}\Delta\delta_{12} + K_{13}\Delta\delta_{13} + K_{14}\Delta\delta_{14}$$
$$0 = -K_{21}\Delta\delta_{12} + K_{22}\Delta\delta_{23} + K_{24}\Delta\delta_{24}$$
$$0 = -K_{31}\Delta\delta_{13} - K_{32}\Delta\delta_{23} + K_{34}\Delta\delta_{34}$$
$$\Delta P_4 = -K_{41}\Delta\delta_{14} - K_{42}\Delta\delta_{24} - K_{43}\Delta\delta_{34}$$

Assuming $E_1$ leads $E_2$ in phase, $E_2$ leads $E_3$, and $E_3$ leads $E_4$

$$\Delta\delta_{13} = \Delta\delta_{12} + \Delta\delta_{23}$$
$$\Delta\delta_{14} = \Delta\delta_{12} + \Delta\delta_{23} + \Delta\delta_{34}$$
$$\Delta\delta_{24} = \Delta\delta_{23} + \Delta\delta_{34}$$

The power increment equations can be expressed in terms of $\Delta\delta_{12}$, $\Delta\delta_{23}$, and $\Delta\delta_{34}$.

$$\Delta P_1 = (K_{12}+K_{13}+K_{14})\Delta\delta_{12} + (K_{13}+K_{14})\Delta\delta_{23} + K_{14}\Delta\delta_{34}$$
$$0 = -K_{21}\Delta\delta_{12} + (K_{22}+K_{24})\Delta\delta_{23} + K_{24}\Delta\delta_{34}$$
$$0 = -K_{31}\Delta\delta_{12} - (K_{31}+K_{32})\Delta\delta_{23} + K_{34}\Delta\delta_{34}$$
$$\Delta P_4 = -K_{41}\Delta\delta_{12} - (K_{41}+K_{42}+K_{43})\Delta\delta_{23} - (K_{41}+K_{42}+K_{43})\Delta\delta_{34}$$

Using the 2 equations for $\Delta P_2$ and $\Delta P_3$ and the relation

$$\Delta\delta_{14} = \Delta\delta_{12} + \Delta\delta_{23} + \Delta\delta_{34}$$

the $\Delta$-angles are expressed in terms of $\Delta\delta_{14}$ by means of determinants.

Let

$$A = \begin{vmatrix} 1 & 1 & 1 \\ -K_{21} & (K_{22}+K_{24}) & K_{24} \\ -K_{31} & -(K_{31}+K_{32}) & K_{34} \end{vmatrix}$$

Then

$$\Delta\delta_{12} = \frac{\begin{vmatrix} (K_{22}+K_{24}) & K_{24} \\ -(K_{31}+K_{32}) & K_{34} \end{vmatrix}}{A}\,\Delta\delta_{14}$$

$$\Delta\delta_{23} = -\frac{\begin{vmatrix} -K_{21} & K_{24} \\ -K_{31} & K_{34} \end{vmatrix}}{A}\,\Delta\delta_{14}$$

$$\Delta\delta_{24} = \frac{\begin{vmatrix} -K_{21} & (K_{23}+K_{24}) \\ -K_{41} & -(K_{41}+K_{43}) \end{vmatrix}}{A} \Delta\delta_{14}$$

Let the ratios of the $\Delta$-angle to $\Delta\delta_{14}$ be expressed by $B$ with the corresponding double subscript, then

$$\Delta\delta_{12} = B_{12}\Delta\delta_{14}$$
$$\Delta\delta_{23} = B_{23}\Delta\delta_{14}$$
$$\Delta\delta_{24} = B_{24}\Delta\delta_{14}$$

Substituting these values in the expressions for $\Delta P_1$ and $\Delta P_4$,

$$\Delta P_1 = [(K_{12}+K_{13}+K_{14})B_{12} + (K_{13}+K_{14})B_{23} + K_{14}B_{24}]\Delta\delta_{14}$$
$$\Delta P_4 = -[K_{41}B_{12} + (K_{41}+K_{42})B_{23} + (K_{41}+K_{42}+K_{43})B_{24}]\Delta\delta_{14}$$

These are the incremental power equations for machines 1 and 4, assuming that the other machines hold constant power. In a similar way expressions for incremental power may be developed for other pairs of machines of a 4-machine system or for systems of more than 4 machines.

## Appendix II—5 Machines

By a procedure similar to that for 4 machines given in Appendix I, the test equations for machines 1 and 5 are obtained for a system of 5 machines.

$$\Delta P_1 = [(K_{12}+K_{13}+K_{14}+K_{15})B_{12} + (K_{13}+K_{14}+K_{15})B_{23} + (K_{14}+K_{15})B_{24} + K_{15}B_{25}]\Delta\delta_{12}$$
$$\Delta P_5 = -[K_{51}B_{12} + (K_{51}+K_{52})B_{23} + (K_{51}+K_{52}+K_{53})B_{24} + (K_{51}+K_{52}+K_{53}+K_{54})B_{25}]\Delta\delta_{12}$$

where

$$A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ -K_{21} & (K_{23}+K_{24}+K_{25}) & (K_{24}+K_{25}) & K_{25} \\ -K_{31} & -(K_{31}+K_{32}) & (K_{34}+K_{35}) & K_{35} \\ -K_{41} & -(K_{41}+K_{42}) & -(K_{41}+K_{42}+K_{43}) & K_{45} \end{vmatrix}$$

$$B_{12} = \frac{\begin{vmatrix} (K_{23}+K_{24}+K_{25}) & (K_{24}+K_{25}) & K_{25} \\ -(K_{31}+K_{32}) & (K_{34}+K_{35}) & K_{35} \\ -(K_{41}+K_{42}) & -(K_{41}+K_{42}+K_{43}) & K_{45} \end{vmatrix}}{A}$$

$$B_{23} = -\frac{\begin{vmatrix} -K_{21} & (K_{24}+K_{25}) & K_{25} \\ -K_{31} & (K_{34}+K_{35}) & K_{35} \\ -K_{41} & -(K_{41}+K_{42}+K_{43}) & K_{45} \end{vmatrix}}{A}$$

$$B_{24} = \frac{\begin{vmatrix} -K_{21} & (K_{23}+K_{24}+K_{25}) & K_{25} \\ -K_{31} & -(K_{31}+K_{32}) & K_{35} \\ -K_{41} & -(K_{41}+K_{42}) & K_{45} \end{vmatrix}}{A}$$

$$B_{25} = -\frac{\begin{vmatrix} -K_{21} & (K_{23}+K_{24}+K_{25}) & (K_{24}+K_{25}) \\ -K_{31} & -(K_{31}+K_{32}) & (K_{34}+K_{35}) \\ -K_{41} & -(K_{41}+K_{42}) & -(K_{41}+K_{42}+K_{43}) \end{vmatrix}}{A}$$

## Appendix III—Equivalent Systems

The practical value of the method of testing for stability given in this paper depends upon the ease and degree of accuracy with which the internal voltage, $E_1$, $E_2$, . . . $E_n$ and the transfer impedances $Z_{12}$, $Z_{13}$, . . . $Z_{1n}$, etc., can be determined. Since the amount of labor involved depends upon the number of synchronous machines and shunt impedances in the system, it is important that the number of each be reduced to the minimum consistent with the accuracy required.

### Principles Used in Combining Machines or Machines and Shunts

First: It has been shown[3] that a synchronous machine whose internal vector voltage is constant in magnitude, with an impedance shunt at its terminals, may be replaced by an equivalent synchronous machine, for determining their effect upon the other machines of the system. The impedance of the equivalent machine will be the impedance of the machine and the shunt in parallel; its internal vector voltage will be $\frac{EZ_s}{Z+Z_s}$, where $E$ and $Z$ are the internal vector voltage and impedance, respectively, of the given machine and $Z_s$ is the impedance of the shunt.

When the shunt contains resistance, power out of the equivalent machine (which is the power out of the shunt plus the power out of the given machine) bears no fixed relation to power out of the actual machine. Consequently, although the shunt and the machine can be replaced by an equivalent machine when their effect upon other machines of the system is being considered, they cannot be replaced by an equivalent machine when maximum power out of the machine itself is under consideration or fixed power is maintained.

Should the shunt have no resistance, such as an inductive or capacitive reactance shunt, power out of the equivalent machine will be the same as that out of the actual machine; consequently, maximum power out of the equivalent machine will be maximum power out of the actual machine. The equivalent machine, therefore, may replace the actual machine, even when maximum power out of the machine itself is being considered or fixed power is being maintained.

Second: Two synchronous machines with their terminals at a common point, whose internal voltages are constant in magnitudes and differ by a constant angle as the machines of the system change their relative angular positions, may be replaced by an equivalent synchronous machine without altering the power delivered or received bythe other synchronous machines of the system. The impedance of the equivalent machine will be the parallel value of the impedances of the 2 given machines. Its internal vector voltage will be $\frac{E_1Z_2 + E_2Z_1}{Z_1+Z_2}$, where $E_1$ and $E_2$ are the internal vector voltages of machines 1 and 2, respectively, and $Z_1$ and $Z_2$ are their impedances.

Third: Two synchronous machines with their terminals separated by a series impedance, whose internal vector voltages are constant and equal in magnitude and do not differ in phase as the machines of the system change their relative angular displacements, may be replaced by an equivalent synchronous machine without changing the power delivered or received by the other machines of the system. The equivalent machine will have the same internal voltage as the actual machine, and its impedance will be determined by replacing the "$\Delta$" made up of the impedances of the 2 machines and the series impedance by an equivalent "$Y$."

## Appendix IV—Reactance of an Equivalent Synchronous Condenser

A synchronous condenser with constant field current operating at a given voltage can be replaced *exactly* in steady state stability calculations by an equivalent condenser with constant reactance. If the current in per unit of rated current is plotted versus the terminal voltage in per unit of rated voltage for any constant field current, the negative of the slope of the curve at the given voltage ($-dV/dI$) will be the reactance of the equivalent condenser in per unit based on its rating. This may be shown as follows: Since steady state stability limit is defined as maximum power at the given operating voltage, the equivalent machine is required to replace the actual machine only when operating in the region of normal voltage. If the equivalent machine for a slight change in terminal voltage provides the same current at the same power factor as the actual machine, then it will represent it exactly in steady state stability calculations.

Let

$X$ = reactance of equivalent condenser corresponding to any given field current in per unit based on its rating
$V_t$ = terminal voltage of condenser in per unit of rated voltage
$E_g$ = internal voltage of equivalent condenser in per unit of rated voltage
$I$ = current of condenser in per unit of rated current

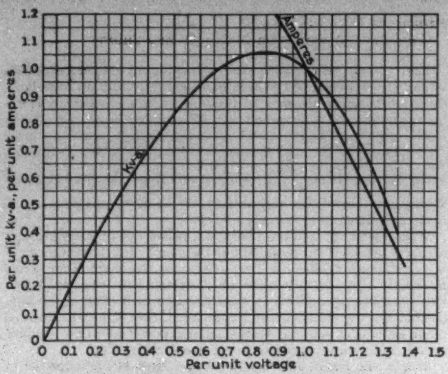



**Fig. 5. Synchronous condenser characteristics for field current corresponding to rated current at rated voltage**

Then

$$E_s = V_t + IX \qquad \text{(D-1)}$$

For a small change in terminal voltage, there will be a small change in current; consequently,

$$E_s = V_t + \Delta V + (I + \Delta I)X \qquad \text{(D-2)}$$

Subtracting eq D-2 from eq D-1 and transposing,

$$X(\Delta I) = -\Delta V$$

$$\therefore X = -\frac{\Delta V}{\Delta I}$$

In the limit

$$X = -\frac{dV}{dI} \qquad \text{(D-3)}$$

The reactance of the equivalent condenser corresponding to any given field current and terminal voltage is, therefore, the negative of the slope of the volt-ampere curve at the given operating voltage.

The volt-ampere characteristic of a condenser may be obtained from its $V$-curves. In Fig. 5 are plotted the kva-volt and ampere-volt characteristic of a synchronous condenser for a field current that will give rated leading current at rated voltage. The direct axis synchronous reactance ($x_d$) of this machine is 1.64. The reactance of the equivalent condenser at rated voltage and rated kva is determined from the slope of the volt-ampere curve at unity voltage. It is

$$X = -\frac{dV}{dI} = 0.56$$

The internal voltage of the equivalent condenser is

$$E_s = E_t + i_d X = 1.0 + 1.0 \times 0.56 = 1.56$$

In a manner similar to this, the reactance of the equivalent condenser for any particular field excitation and terminal voltage may be determined from the volt-ampere characteristics of the condenser.

## Appendix V—
## Induction Machines as Fixed Power Machines

An induction machine can be represented[8] by the equivalent circuit given in Fig. 6A or by the less exact one of Fig. 6B. If a synchronous machine of the proper reactance $X$, and internal voltage $E$ replace that part of the equivalent induction machine circuit to the right of point $O$ in Fig. 6A, or to the right of point $O'$ in Fig. 6B, the synchronous machine together with the impedance network to the left of $O$ or $O'$ will represent correctly the induction machine with fixed power on its shaft for small changes in its terminal voltage.

The reactance of the synchronous machine is determined in the following way: The power at $O$, Fig. 6A, is the power on the shaft of the motor; this power must remain constant with changes in current and voltage at $O$. With a drop in voltage at $O$, there will be a change in current and in slip for constant power; there will be also a change in power factor at $O$. Knowing the constants of the equivalent circuit for the induction machine, it is possible to determine the power factor and voltage at $O$ corresponding to any given shaft power and terminal voltage. These may be determined more readily at $O'$ in the approximate induction machine circuit given in Fig. 6B. From either circuit, it is possible from given initial conditions of terminal voltage, $V_1$, and power on the shaft of the machine, $P$, to calculate initial voltage and power factor at $O$ or $O'$. Change of power factor with change in voltage at $O$ and $O'$, shaft power remaining constant, also may be determined.

The equivalent fixed power synchronous machine for small change in voltage at $O$ or $O'$ must give the same current at the same power factor as will be obtained from the standard equivalent induction machine circuit.

Let

$X$ = reactance of equivalent synchronous machine to be connected at $O$ or $O'$
$E$ = internal voltage of equivalent synchronous machine
$\theta$ = $\cos^{-1}$ (power factor at $O$ or $O'$)
$P$ = power on shaft of machine
$V$ = voltage at $O$ or $O'$ corresponding to initial terminal voltage $V_1$ and shaft power $P$

Then

$$I_2 = \frac{P}{V}(1 \mp j \tan\theta)$$

The negative sign is used with $\tan\theta$ when the power factor at $O$ is lagging, the plus sign when leading. For lagging power factor,

$$E = V - jXI_2 = V - j\frac{XP}{V}(1 - j\tan\theta)$$

$$= (V - \frac{XP}{V}\tan\theta) - j\frac{XP}{V}$$

$$|E|^2 = V^2 - 2XP\tan\theta + \frac{X^2P^2}{V^2}(1 + \tan^2\theta)$$

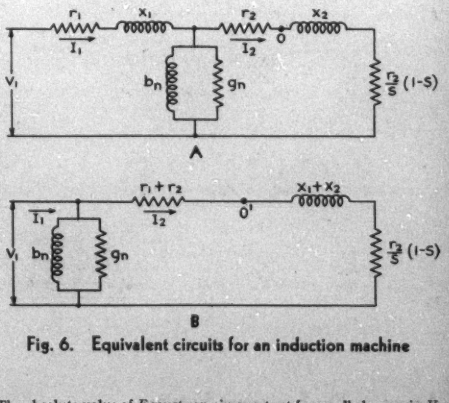



**Fig. 6. Equivalent circuits for an induction machine**

The absolute value of $E$ must remain constant for small changes in $V$

$$\therefore \frac{d|E|^2}{dV} = 0 = 2V - 2XP\frac{d\tan\theta}{dV} + \frac{X^2P^2}{V^4}\left[2V^2\tan\theta\frac{d\tan\theta}{dV} - 2V(1 + \tan^2\theta)\right]$$

Transposing and dividing through by $-2V$, and letting $\frac{d\tan\theta}{dV} = n$

$$X^2\left(\frac{P}{V^2}\right)^2[\tan^2\theta-(nV)\tan\theta+1]+X\left(\frac{P}{V^2}\right)(nV)-1=0$$

Solving,

$$X=\frac{V^2}{P}\left[\frac{-nV+\sqrt{(nV)^2+4\tan^2\theta-(nV)\tan\theta+1}}{2(\tan^2\theta-\tan\theta(nV)+1)}\right] \quad \text{(E-1)}$$

If power is constant at the terminals of the induction machine, or shaft power is constant but losses are neglected, eq E-1 will give the reactance of the equivalent synchronous machine which replaces the induction machines at its terminals, point $O$ being at the terminals. For the usual operating conditions, $\tan\theta$ then will decrease with a decrease in voltage and $n=\frac{\mathrm{d}\tan\theta}{\mathrm{d}V}$ will be positive. If $\tan\theta$ increases with a decrease in voltage, $n$ will be negative.

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# Steady State Stability of Composite Systems


**Composite loads and induction machines are included in the study of steady state stability characteristics of power systems reported in this paper. Reactive power characteristics must be considered when induction machines are involved, in addition to the real power characteristics which are considered for synchronous machine groups. A determination of the synchronizing power coefficients under various operating conditions, as described herein, provides a comparatively simple method of determining the safe and dangerous regions of steady state operation.**



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A MEASURE of the steady state stability of a synchronous machine, or an equivalent synchronous machine group, can be obtained from equations for real power alone, by calculating the rate of change of electrical power with change in relative angular displacement. This factor, hereinafter called the synchronizing power coefficient $\frac{\mathrm{d}P}{\mathrm{d}\delta}$, is described and discussed in the companion paper,[1] and for a given operating condition its magnitude and sign indicates the degree of stability.

When machines other than the synchronous type are studied, most important of which is the induction machine, the reactive power characteristics must be considered in addition to the real power characteristics. The real power-voltage and reactive power-voltage characteristics of any load or machine are sufficient to determine its stabilizing influence on the remaining machines. Using a method of analysis similar to that of the companion paper, expressions based upon real and reactive power in a network are derived in Part I of the present paper, for the synchronizing power coefficient when the system contains composite loads which include induction machines. In Part II is presented a method for determining whether or not an induction machine can operate at the assumed or specific operating condition; this makes it possible to determine the degree of asynchronous stability of an induction machine. In the appendixes are presented methods of determining for a composite load which may include synchronous machines, impedance loads, and induction machines, the real and reactive power-voltage characteristics, $\frac{\mathrm{d}P}{\mathrm{d}E}$ and $\frac{\mathrm{d}Q}{\mathrm{d}E}$. The general equations given may be applied for any combination of loads and machines of a system, their application not being limited to the specific examples presented in the paper.


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1. For all numbered references see list at end of paper.

## Part I—Synchronizing Power Coefficients of Systems Involving Composite Loads

A composite load consists of a large number of individual loads which would involve a great deal of labor if treated separately when studying the stability characteristics of the major machine groups. The effect of composite loads on the system may be

evaluated by determining their real power and reactive power characteristics with change in voltage. Synchronous converters, heating and lighting loads and loads due to transformer magnetizing currents can be represented approximately by equivalent shunt impedances. The real power-voltage and reactive power-voltage characteristics of an impedance load can be calculated as described in Appendix II. Appendixes III and IV give methods for determining the real and reactive power characteristics of induction and synchronous machines. Knowing the characteristics and proportions of the various types of load, the real and reactive power-voltage characteristics of each composite load can be determined.

The nomenclature used in this paper is as follows. All quantities are expressed in per unit on the system kilovoltampere and voltage bases:

- $E$ = voltage
- $I$ = current
- $P$ = real power into the system (normal power for a generator will therefore be positive, for a motor negative)
- $Q$ = reactive power (reactive power into the system from an overexcited machine is negative)
- $s$ = slip (positive for a motor, negative for a generator)
- $r$ = resistance
- $x$ = reactance
- $Z$ = impedance
- $Z_{jj}$ = driving point impedance from point $j$
- $Z_{jk} = Z_{kj}$ = transfer impedance between points $j$ and $k$
- $\theta_{jj}$ = impedance angle of $Z_{jj}$
- $\theta_{jk} = \theta_{kj}$ = impedance angle of $Z_{jk}$ or $Z_{kj}$
- $\alpha_{jj} = 90 \text{ deg} - \theta_{jj}$
- $\alpha_{jk} = 90 \text{ deg} - \theta_{jk}$
- $\delta_{jk} = \delta_j - \delta_k$, angular displacement between voltages $E_j$ and $E_k$
- $\alpha_n$ = per unit governor regulation of machine group $n$
- $C_n$ = per unit generator kva of machine group $n$

Assuming that the characteristics of the composite loads are known or obtainable, the following general equations may be written for $n$ synchronous machine groups and $z$ composite loads, where the numerical and $n$th subscripts refer to synchronous machines or machine groups, and the letters to composite loads. (See Appendix I for derivation of power equations.)

$$
\left.\begin{aligned}
P_1 &= H_{11} + H_{12} \dots + H_{1n} + H_{1a} \dots + H_{1z} \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
P_n &= H_{n1} + H_{n2} \dots + H_{n(n-1)} + H_{nn} \dots + H_{nz} \\
P_a &= H_{a1} + H_{a2} \dots + H_{an} + H_{ab} \dots + H_{az} \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
P_z &= H_{z1} + H_{z2} \dots + H_{zn} + H_{za} \dots + H_{zz} \\
Q_a &= -K_{a1} + K_{a2} \dots + K_{an} + K_{ab} \dots + K_{az} \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
Q_z &= -K_{z1} + K_{z2} \dots + K_{zn} + K_{za} \dots + K_{zz}
\end{aligned}\right\} \quad (1)
$$

Since $E_1, E_2, \dots E_n$ are constant, eq 1 yields for small changes,

$$
\left.\begin{aligned}
dP_1 &= f(d\delta_{12} \dots d\delta_{1n}, d\delta_{1a} \dots d\delta_{1z}, dE_a \dots dE_z) \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
dP_n &= f(d\delta_{n1} \dots d\delta_{n(n-1)}, d\delta_{nn} \dots d\delta_{nz}, dE_a \dots dE_z) \\
dP_a &= f(d\delta_{a1} \dots d\delta_{an}, d\delta_{ab} \dots d\delta_{az}, dE_a \dots dE_z) \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
dP_z &= f(d\delta_{z1} \dots d\delta_{zn}, d\delta_{za} \dots d\delta_{zz}, dE_a \dots dE_z) \\
dQ_a &= f(d\delta_{a1} \dots d\delta_{an}, d\delta_{ab} \dots d\delta_{az}, dE_a \dots dE_z) \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
dQ_z &= f(d\delta_{z1} \dots d\delta_{zn}, d\delta_{za} \dots d\delta_{zz}, dE_a \dots dE_z)
\end{aligned}\right\} \quad (2)
$$

The following relations exist between the incremental angles:

$$
\left.\begin{aligned}
d\delta_{nn} &= -d\delta_{nn} \\
d\delta_{12} &= d\delta_{13} + d\delta_{23} \\
d\delta_{14} &= d\delta_{13} + d\delta_{34}, \text{ etc.}
\end{aligned}\right\} \quad (3)
$$

Equations 2 and 3 may be solved simultaneously for any particular case to determine any synchronizing power coefficient. The use of these general equations can best be demonstrated by 2 relatively simple cases.

## Case 1—Two Synchronous Machine Groups and One Composite Load

For this case,

$$
\left.\begin{aligned}
P_1 &= H_{11} + H_{12} + H_{1a} \\
P_2 &= H_{22} + H_{21} + H_{2a} \\
P_a &= H_{aa} + H_{a1} + H_{a2} \\
Q_a &= -K_{aa} + K_{a1} + K_{a2}
\end{aligned}\right\} \quad (4)
$$

$$
0 = d\delta_{12} + d\delta_{2a} - d\delta_{1a} \quad (5)
$$

Since

$$
\frac{\partial H_{jk}}{\partial \delta_{jk}} = K_{jk} \qquad \frac{\partial K_{jk}}{\partial \delta_{jk}} = -H_{jk}
$$

$$
\frac{\partial H_{ja}}{\partial E_a} = \frac{H_{ja}}{E_a} \qquad \frac{\partial K_{ja}}{\partial E_a} = \frac{K_{ja}}{E_a}
$$

$$
\frac{\partial H_{aa}}{\partial E_a} = \frac{2H_{aa}}{E_a} \qquad \frac{\partial K_{aa}}{\partial E_a} = \frac{2K_{aa}}{E_a}
$$

for small changes, eq 4 yields

$$
\left.\begin{aligned}
dP_1 &= K_{12}d\delta_{12} + K_{1a}d\delta_{1a} + H_{1a}\frac{dE_a}{E_a} \\
dP_2 &= K_{21}d\delta_{21} + K_{2a}d\delta_{2a} + H_{2a}\frac{dE_a}{E_a} \\
dP_a &= K_{a1}d\delta_{a1} + K_{a2}d\delta_{a2} + (2H_{aa} + H_{a1} + H_{a2})\frac{dE_a}{E_a} \\
dQ_a &= -H_{a1}d\delta_{a1} - H_{a2}d\delta_{a2} + (-2K_{aa} + K_{a1} + K_{a2})\frac{dE_a}{E_a}
\end{aligned}\right\} \quad (6)
$$

Replacing $d\delta_{21}$, $d\delta_{a1}$ and $d\delta_{a2}$ in eq 6 by $-d\delta_{12}$, $-d\delta_{1a}$ and $-d\delta_{2a}$, respectively, and rearranging,

$$
dP_1 = K_{11}d\delta_{12} + K_{1a}d\delta_{1a} + H_{1a}\frac{dE_a}{E_a} \quad (7)
$$

$$
dP_2 = -K_{21}d\delta_{12} + K_{2a}d\delta_{2a} + H_{2a}\frac{dE_a}{E_a} \quad (8)
$$

$$
0 = -K_{a1}d\delta_{1a} - K_{a2}d\delta_{2a} + A_a\frac{dE_a}{E_a} \quad (9)
$$

$$
0 = H_{a1}d\delta_{1a} + H_{a2}d\delta_{2a} + B_a\frac{dE_a}{E_a} \quad (10)
$$

where

$$
A_a = 2H_{aa} + H_{a1} + H_{a2} - \frac{dP_a}{dE_a}E_a
$$

$$
B_a = -2K_{aa} + K_{a1} + K_{a2} - \frac{dQ_a}{dE_a}E_a
$$

Solving eqs 5, 7, 9, and 10 simultaneously, the synchronizing power coefficient for machine 1 is

$$
\frac{dP_1}{d\delta_{12}} = \frac{\begin{vmatrix} K_{1a} + K_{12} & -K_{12} & H_{1a} \\ -K_{a1} & -K_{a2} & A_a \\ H_{a1} & H_{a2} & B_a \end{vmatrix}}{\begin{vmatrix} -K_{a1} & -K_{a2} & A_a \\ H_{a1} & H_{a2} & B_a \\ 1 & -1 & 0 \end{vmatrix}} \quad (11)
$$

Solving eqs 5, 8, 9, and 10 simultaneously, the synchronizing power coefficient for machine 2 is

$$
\frac{dP_2}{d\delta_{21}} = \frac{\begin{vmatrix} -K_{21} & K_{2a} + K_{21} & H_{2a} \\ -K_{a1} & -K_{a2} & A_a \\ H_{a1} & H_{a2} & B_a \end{vmatrix}}{\begin{vmatrix} -K_{a1} & -K_{a2} & A_a \\ H_{a1} & H_{a2} & B_a \\ -1 & 1 & 0 \end{vmatrix}} \quad (12)
$$

Equations 11 and 12 can be used as a measure of the ability of machines 1 and 2 to remain in static equilibrium for any given set of load conditions. That is, the larger the value of $dP/d\delta$ when positive, the greater will be the forces developed to keep the particular machine in question in synchronism. It will be noted that a change in the characteristics of the composite load affects only the terms $A_a$ and $B_a$. This makes it possible to readily investigate the stability of machines 1 and 2 with change in the composite load characteristics.

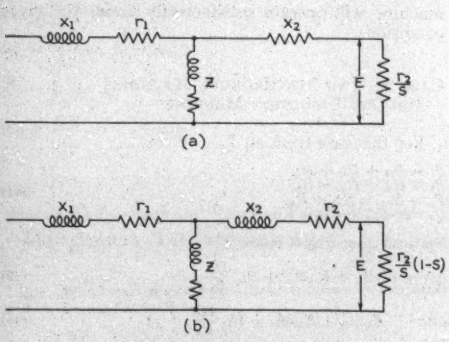


**Fig. 1. Equivalent circuits of induction machines**

If the composite load consists of an individual induction machine or a group of induction machines and $E_a$ is the voltage across the equivalent secondary load resistance, $\frac{r_2}{s}$, of the induction machine equivalent circuit as in Fig. 1(*a*),

$$\frac{dQ_a}{dE_a} = 0 \text{ and } \frac{dP_a}{dE_a} = -\frac{2s^2nE_a}{r_2(sn - s + 1)} \text{ (Derived in Appendix VI.)}$$

Where $n$ is defined by the equation,

$$T_m \propto (1 - s)^n$$

for the mechanical shaft torque of the induction machine. For example, $n = 0$ and $\frac{dP_a}{dE_a} = 0$ for a constant shaft torque load.

### Case 2—Three Synchronous Machine Groups and One Composite Load

For this case,

$$\left.\begin{aligned} P_1 &= H_{11} + H_{12} + H_{13} + H_{1a} \\ P_2 &= H_{22} + H_{21} + H_{23} + H_{2a} \\ P_3 &= H_{33} + H_{31} + H_{32} + H_{3a} \\ P_a &= H_{aa} + H_{a1} + H_{a2} + H_{a3} \\ Q_a &= -K_{aa} + K_{a1} + K_{a2} + K_{a3} \end{aligned}\right\} \quad (13)$$

For small changes eq 13 gives,

$$dP_1 = K_{12}d\delta_{12} + K_{13}d\delta_{13} + K_{1a}d\delta_{1a} + H_{1a}\frac{dE_a}{E_a} \quad (14)$$

$$dP_2 = -K_{21}d\delta_{12} + K_{22}d\delta_{23} + K_{2a}d\delta_{2a} + H_{2a}\frac{dE_a}{E_a} \quad (15)$$

$$dP_3 = -K_{31}d\delta_{13} - K_{32}d\delta_{23} + K_{3a}d\delta_{3a} + H_{3a}\frac{dE_a}{E_a} \quad (16)$$

$$0 = -K_{a1}d\delta_{1a} - K_{a2}d\delta_{2a} - K_{a3}d\delta_{3a} + A_a\frac{dE_a}{E_a} \quad (17)$$

$$0 = H_{a1}d\delta_{1a} + H_{a2}d\delta_{2a} + H_{a3}d\delta_{3a} + B_a\frac{dE_a}{E_a} \quad (18)$$

where for this case,

$$A_a = 2H_{aa} + H_{a1} + H_{a2} + H_{a3} - \frac{dP_a}{dE_a}E_a \quad (19)$$

$$B_a = -2K_{aa} + K_{a1} + K_{a2} + K_{a3} - \frac{dQ_a}{dE_a}E_a \quad (20)$$

The independent relations between the differential angles $d\delta_1$, $d\delta_2$, $d\delta_3$, and $d\delta_a$ are

$$\left.\begin{aligned} d\delta_{13} &= d\delta_{12} + d\delta_{23} \\ d\delta_{1a} &= d\delta_{13} + d\delta_{3a} \\ d\delta_{2a} &= d\delta_{23} + d\delta_{3a} \end{aligned}\right\} \quad (21)$$

If machines 1 and 2 divide their increments in load in the ratio

$$\frac{dP_2}{dP_1} = R,$$

$$dP_2 = RdP_1 \quad (22)$$

The ratio $R$ may be determined by eq 27*a* of Appendix V.

$$R = \frac{C_2\alpha_1}{C_1\alpha_2} \quad (23)$$

If machine 2 is a constant power machine, $R = 0$.

Equations 14, 15, 17, 18, 21, and 22 may be solved simultaneously to determine $dP_1/d\delta_{13}$ or $dP_2/d\delta_{23}$; and 14–18, 21, and 22 for $dP_3/d\delta_{31}$ or $dP_3/d\delta_{32}$. The evaluation of the resulting determinants can be accomplished numerically without a great deal of difficulty for any given problem.

### Additional Cases

Additional cases for any combination of machines and loads may be worked out by applying the general equations and the method outlined above.

## Part II—Asynchronous Stability of Induction Machines

In certain cases it may be necessary to determine whether or not an induction machine when connected to a network, will operate at the specified operating conditions. An induction motor or generator will operate satisfactorily[6,7] if

$$\frac{dT_e}{ds} > \frac{dT_m}{ds} \quad (24)$$

where

$T_e$ = electrical shaft torque
$T_m$ = mechanical shaft torque
$s$ = slip

The following analysis is based upon the assumption that the electrical shaft torque of an induction machine can be expressed by either of the 2 equations

$$T_e = \frac{I^2r_2}{s} \quad (25)$$

and

$$T_e = EI \quad (26)$$

where $I$ is the secondary current, $r_2$ the secondary resistance, $s$ the slip, and $E$ the voltage across the equivalent secondary load resistance $r_2/s$. See Fig. 1(*a*).

Equations 25 and 26 yield for small changes,

$$dT_e = 2EdI - \frac{I^2 r_2}{s^2}\,ds \tag{27}$$

$$dT_e = EdI + IdE \tag{28}$$

Solving eqs 27 and 28 for $ds$ in terms of $T_e$ and $dE$,

$$ds = \frac{s^2}{I^2 r_2}\,[dT_e - 2IdE] \tag{29}$$

therefore,

$$\frac{dT_e}{ds} = \frac{I^2 r_2}{s^2}\left[\frac{\dfrac{dT_e}{dE}}{\dfrac{dT_e}{dE} - 2I}\right] \tag{30}$$

In the above analysis power into the induction motor has been considered positive; but, since the convention has been adopted that power into a machine from the system is negative, eq 30 becomes, in terms of network power,

$$\frac{dT_e}{ds} = \frac{I^2 r_2}{s^2}\left[\frac{\dfrac{dP_a}{dE_a}}{\dfrac{dP_a}{dE_a} + \dfrac{2E_a s^2}{r_2}}\right] \tag{31}$$

$P_a$ and $E_a$ are power from point $a$ into the network and voltage at point $a$, respectively, $E_a$ being the voltage across the equivalent induction motor load resistance, $r_2/s$.

Substituting eq 31 and 29*a* of Appendix VI in eq 24 and simplifying, the criterion for satisfactory operation of the induction machine is

$$\left[\frac{\dfrac{dP_a}{dE_a}}{\dfrac{dP_a}{dE_a} + \dfrac{2E_a s^2}{r_2}}\right] > -\frac{sn}{(1-s)} \tag{32}$$

Where $n$ is defined by the equation,

$$T_m \propto (1-s)^n \tag{33}$$

$\dfrac{dP_a}{dE_a}$ in eq 32 depends upon the connected system and can be determined for any given case by means of the general equations for real and reactive power flow. Two relatively simple cases will be used to illustrate the method of determining $\dfrac{dP_a}{dE_a}$.

### Case 1—One Synchronous Machine and One Induction Machine

For this case, from eq 1,

$$\left.\begin{aligned} P_1 &= H_{11} + H_{1a} \\ P_a &= H_{aa} + H_{a1} \\ Q_a &= -K_{aa} + K_{a1} \end{aligned}\right\} \tag{34}$$

For small changes eq 34 yields,

$$dP_1 = K_{1a}d\delta_{1a} + H_{1a}\frac{dE_a}{E_a} \tag{35}$$

$$dP_a = -K_{a1}d\delta_{1a} + (2H_{aa} + H_{a1})\frac{dE_a}{E_a} \tag{36}$$

$$dQ_a = H_{a1}d\delta_{1a} + (-2K_{aa} + K_{a1})\frac{dE_a}{E_a} \tag{37}$$

Since $E_a$ is the voltage across the secondary load resistance of the induction machine, $\dfrac{dQ_a}{dE_a} = 0$. Equations 36 and 37 therefore can be written as follows,

$$\frac{dP_a}{dE_a} = -K_{a1}\frac{d\delta_{1a}}{dE_a} + \frac{2H_{aa} + H_{a1}}{E_a} \tag{38}$$

$$0 = H_{a1}\frac{d\delta_{1a}}{dE_a} + \frac{-2K_{aa} + K_{a1}}{E_a} \tag{39}$$

Solving eqs 38 and 39 simultaneously for $\dfrac{dP_a}{dE_a}$ in terms of the $H$ and $K$ constants,

$$\frac{dP_a}{dE_a} = \frac{1}{E_a}\left[\frac{K_{a1}}{H_{a1}}(-2K_{aa} + K_{a1}) + (2H_{aa} + H_{a1})\right] \tag{40}$$

For any given set of operating conditions $dP_a/dE_a$ may be determined from eq 40 and then substituted in eq 33 to determine whether or not the induction machine will operate satisfactorily under the given conditions.

### Case 2—Two Synchronous Machines and One Induction Machine

For this case from eq 1,

$$\left.\begin{aligned} P_1 &= H_{11} + H_{12} + H_{1a} \\ P_2 &= H_{21} + H_{22} + H_{2a} \\ P_a &= H_{aa} + H_{a1} + H_{a2} \\ Q_a &= -K_{aa} + K_{a1} + K_{a2} \end{aligned}\right\} \tag{41}$$

For small changes eq 41 yields,

$$dP_1 = K_{12}d\delta_{12} + K_{1a}d\delta_{1a} + H_{1a}\frac{dE_a}{E_a} \tag{42}$$

$$dP_2 = -K_{21}d\delta_{12} + K_{2a}d\delta_{2a} + H_{2a}\frac{dE_a}{E_a} \tag{43}$$

$$dP_a = -K_{a1}d\delta_{1a} - K_{a2}d\delta_{2a} + (2H_{aa} + H_{a1} + H_{a2})\frac{dE_a}{E_a} \tag{44}$$

$$dQ_a = H_{a1}d\delta_{1a} + H_{a2}d\delta_{2a} + (-2K_{aa} + K_{a1} + K_{a2})\frac{dE_a}{E_a} \tag{45}$$

Also

$$d\delta_{1a} = d\delta_{12} + d\delta_{2a} \tag{46}$$

$$dQ_a = 0 \tag{47}$$

$$dP_2 = RdP_1 \tag{48}$$

where $R$ is the ratio of the division of the increment in power between the two synchronous machine groups, and is approximately equal to $\dfrac{C_2}{C_1}\dfrac{\alpha_2}{\alpha_1}$. (See Appendix V.) If synchronous machine 2 is a fixed power synchronous machine, $R = 0$.

Equations 42 to 48 can be solved simultaneously to obtain $\dfrac{dP_a}{dE_a}$.

### Additional Cases

Additional cases of any combination of synchronous machines, induction machines, or composite loads, may be worked out by applying the methods presented above.

## Appendix I—Equations for Real and Reactive Power

Equations for the real[3] and reactive power[3] in a balanced network may be derived as follows,

$$\bar{I}_j = \text{current at point } j = \frac{\bar{E}_j}{\bar{Z}_{jj}} - \frac{\bar{E}_k}{\bar{Z}_{jk}} \cdots - \frac{\bar{E}_n}{\bar{Z}_{jn}} \tag{1a}$$

$P_j$ = real power = $\overline{E}_j \cdot \overline{I}_j$

$$= \frac{E_j^2}{Z_{jj}} \cos \theta_{jj} - \frac{E_j E_k}{Z_{jk}} \cos (\delta_{jk} + \theta_{jk}) \ldots - \frac{E_j E_n}{Z_{jn}} \cos (\delta_{jn} + \theta_{jn})$$

$$= \frac{E_j^2}{Z_{jj}} \sin \alpha_{jj} + \frac{E_j E_k}{Z_{jk}} \sin (\delta_{jk} - \alpha_{jk}) \ldots + \frac{E_j E_n}{Z_{jn}} \sin (\delta_{jn} - \alpha_{jn})$$

$$= H_{jj} + H_{jk} \ldots . + H_{jn} \tag{2a}$$

$Q_j$ = reactive power = scalar value of $\overline{E}_j \times \overline{I}_j$

$$= - \frac{E_j^2}{Z_{jj}} \cos \alpha_{jj} + \frac{E_j E_k}{Z_{jk}} \cos (\delta_{jk} - \alpha_{jk}) \ldots + \frac{E_j E_n}{Z_{jn}} \cos (\delta_{jn} - \alpha_{jn})$$

$$= -K_{jj} + K_{jk} \ldots . + K_{jn} \tag{3a}$$

where

$$H_{jj} = \frac{E_j^2}{Z_{jj}} \sin \alpha_{jj}$$

$$H_{jk} = \frac{E_j E_k}{Z_{jk}} \sin (\delta_{jk} - \alpha_{jk})$$

$$K_{jj} = \frac{E_j^2}{Z_{jj}} \cos \alpha_{jj}$$

$$K_{jk} = \frac{E_j E_k}{Z_{jk}} \cos (\delta_{jk} - \alpha_{jk})$$

## Appendix II—Real and Reactive Power-Voltage Characteristics of an Impedance Load

The real and reactive power into the system from an impedance load can be expressed as follows,

$$P_a = - \frac{E_a^2}{Z} \cos \theta \tag{4a}$$

$$Q_a = \frac{E_a^2}{Z} \sin \theta \tag{5a}$$

where

$Z$ = impedance of load
$\theta$ = impedance angle of $Z$

Therefore,

$$\frac{dP_a}{dE_a} = - \frac{2E_a}{Z} \cos \theta = \frac{2P_a}{E_a} \tag{6a}$$

$$\frac{dQ_a}{dE_a} = \frac{2E_a}{Z} \cos \theta = \frac{2Q_a}{E_a} \tag{7a}$$

Equations 6a and 7a define the real and reactive power-voltage characteristics of an impedance load.

## Appendix III—Real and Reactive Power-Voltage Characteristics of Induction Machine

An induction machine may be represented[4] by the circuit of Fig. 2, where $E_a$ is the terminal voltage and $E_b$ the voltage across the equivalent shaft load resistance. From eq 1, the following relations can be written,

$$\left.\begin{aligned} -P_a &= H_{aa} + H_{ab} \\ -Q_a &= -K_{aa} + K_{ab} \\ P_b &= H_{bb} + H_{ba} \\ Q_b &= -K_{bb} + K_{ba} \end{aligned}\right\} \tag{8a}$$

For small changes these equations yield,

$$-dP_a = K_{ab} d\delta_{ab} + (2H_{ab} + H_{aa}) \frac{dE_a}{E_a} + H_{ab} \frac{dE_b}{E_b} \tag{9a}$$

$$-dQ_a = -H_{ab} d\delta_{ab} + (-2K_{aa} + K_{ab}) \frac{dE_a}{E_a} + K_{ab} \frac{dE_b}{E_b} \tag{10a}$$

$$dP_b = -K_{ba} d\delta_{ab} + (2H_{bb} + H_{ba}) \frac{dE_b}{E_b} + H_{ba} \frac{dE_a}{E_a} \tag{11a}$$

$$dQ_b = H_{ba} d\delta_{ab} + (-2K_{bb} + K_{ba}) \frac{dE_b}{E_b} + K_{ba} \frac{dE_a}{E_a} \tag{12a}$$

Since $Q_b$ is constant and equal to zero, $dQ_a = 0$. Substituting this value in eq 11a and solving eq 11a and 12a simultaneously with 9a and 10a, respectively, there results,

$$\frac{dP_a}{dE_a} = - \frac{(2H_{aa} + H_{ab}) + AK_{ab} + BH_{ab}}{E_a} \tag{13a}$$

$$\frac{dQ_a}{dE_a} = + \frac{(2K_{aa} - K_{ab}) + AH_{ab} - BK_{ab}}{E_a} \tag{14a}$$

where

$$A = \frac{2H_{bb}K_{ba} + 2K_{bb}H_{ba} - K_{ba}E_b \dfrac{dP_b}{dE_b}}{+K_{ba}(2K_{bb} - K_{ba}) - H_{ba}\left(2H_{bb} + H_{ba} - E_b \dfrac{dP_b}{dE_b}\right)} \tag{15a}$$

$$B = \frac{K_{ba}^2 + H_{ba}^2}{+K_{ba}(2K_{bb} - K_{ba}) - H_{ba}\left(2H_{bb} + H_{ba} - E_b \dfrac{dP_b}{dE_b}\right)} \tag{16a}$$

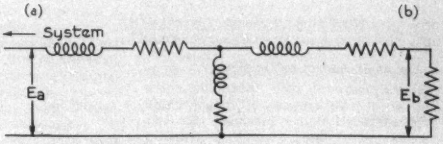


Fig. 2. Equivalent circuit of an induction machine

Equations 13a and 14a define the real and reactive power-voltage characteristics of the induction machine. $\frac{dP_b}{dE_b}$ depends upon the mechanical shaft load of the induction machine and as shown in Appendix VI can be expressed in terms of the initial quantities as follows, when $E_b$ is the voltage across the equivalent secondary load resistance $r_2/s$,

$$\frac{dP_b}{dE_b} = - \frac{2s^2 n E_b}{r_2(sn - s + 1)} \tag{17a}$$

where

$n = 0$ for constant shaft torque
$n = 1$ for a shaft torque which varies directly with the speed.

If $E_b$ is taken as the voltage across the equivalent shaft load resistance, $\frac{r_2}{s}(1 - s)$ instead of across $\frac{r_2}{s}$, $\frac{dP_b}{dE_b} = 0$ for a constant shaft power load. (See Fig. 1(*b*).)

Equations 9a to 12a inclusively, may be used in reducing or simplifying a complicated composite load network, as well as determining the characteristics of induction machines. That is, if $\frac{dP_b}{dE_b}$ and $\frac{dQ_b}{dE_b}$ are known at a point *b* in a network $\frac{dP_a}{dE_a}$ and $\frac{dQ_a}{dE_a}$ at point *a* may be determined.

## Appendix IV—Power-Voltage Characteristics of a Constant Power Synchronous Machine (Saturation and Resistance Neglected)

The steady state vector diagram of a salient pole synchronous machine[4] is shown in Fig. 3. $E_a$ is the terminal voltage and $E_i$ the internal voltage.

From Fig. 3,

$$\bar{E}_a = E_a \cos \delta_{1a} - jE_a \sin \delta_{1a} \tag{18a}$$

$$\dot{i}_a = i_q - ji_d = \frac{E_a \sin \delta_{1a}}{x_q} - j\frac{E_1 - E_a \cos \delta_{1a}}{x_d} \tag{19a}$$

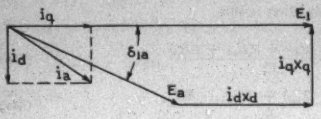


Fig. 3. Vector diagram of salient pole synchronous machine

The real and reactive power into the system may be expressed as

$$P_a = \bar{E}_a \cdot \dot{i} = \frac{E_1 E_a}{x_d} \sin \delta_{1a} + \frac{E_a^2(x_d - x_q)}{2x_d x_q} \sin 2\delta_{1a} \tag{20a}$$

$$Q_a = \text{scalar value of } \bar{E}_a \times \dot{i}_a = -\frac{E_1 E_a}{x_d} \cos \delta_{1a} + E_a^2 \left[\frac{1}{x_d} + \frac{x_d - x_q}{x_d x_q} \sin^2 \delta_{1a}\right] \tag{21a}$$

For small changes and $dP = 0$, eq 20$a$ yields

$$\frac{d\delta_{1a}}{dE_a} = -\frac{[E_1 x_q \sin \delta_{1a} + E_a(x_d - x_q) \sin 2\delta_{1a}]}{E_a[E_1 x_q \cos \delta_{1a} + E_a(x_d - x_q) \cos 2\delta_{1a}]} \tag{22a}$$

For small changes eq 21$a$ yields,

$$\frac{dQ_a}{dE_a} = \frac{1}{x_d x_q}[-E_1 x_q \cos \delta_{1a} + 2E_a x_q + 2E_a(x_d - x_q) \sin^2 \delta_{1a}] + \frac{E_a}{x_d x_q}[E_1 x_q \sin \delta_{1a} + E_a(x_d - x_q) \sin 2\delta_{1a}] \frac{d\delta_{1a}}{dE_a} \tag{23a}$$

Substituting eq 22$a$ in eq 23$a$ and simplifying

$$\frac{dQ_a}{dE_a} = \frac{-E_1^2 x_q^2 + E_1 E_a x_q \cos \delta_{1a}[3x_q - x_d] + 2E_a^2(x_d - x_q)[x_q \cos^2 \delta_{1a} - x_d \sin^2 \delta_{1a}]}{x_d x_q[E_1 x_q \cos \delta_{1a} + E_a(x_d - x_q) \cos 2\delta_{1a}]} \tag{24a}$$

For the case of a cylindrical rotor machine, $x_d = x_q$, and

$$\frac{dQ_a}{dE_a} = \frac{1}{x_d}[2E_a - E_1 \sec \delta_{1a}] \tag{25a}$$

Equations 24$a$ and 25$a$ define the reactive power-voltage characteristics of a salient pole machine and a cylindrical rotor machine, respectively, with constant shaft load when resistance and saturation are neglected.

The real-power voltage characteristic of such machines is

$$\frac{dP}{dE_a} = 0$$

## Appendix V—Proportioning of Load Between Generator Groups With Increase in System Load

When an additional load, $dP$, is placed on the system,[5] the following general equation may be used to determine approximately the load division between the generator groups,

$$dP_n = \frac{\frac{C_n}{\alpha_n}}{\frac{C_1}{\alpha_1} + \frac{C_2}{\alpha_2} \cdots + \frac{C_n}{\alpha_n}} dP \tag{26a}$$

where

$dP_n$ = additional load on machine $n$
$\alpha_1$ = per unit governor regulation of machine group 1
$\alpha_n$ = per unit governor regulation of machine group $n$
$C_1$ = per unit generator capacity of machine group 1
$C_n$ = per unit generator capacity of machine group $n$

The ratio $R$ between the power increment of any 2 generator groups is therefore simply,

$$R = \frac{dP_2}{dP_1} = \frac{C_2 \alpha_1}{C_1 \alpha_2} \tag{27a}$$

## Appendix VI—Real Power-Secondary Voltage Characteristic of Variable Shaft Load Induction Machine

For the case when the mechanical shaft load is a function of the speed,

$$T_m = k(1 - s)^n \tag{28a}$$

and

$$dT_m = -kn(1 - s)^{n-1}\, ds$$

Since,

$$k = \frac{E^2 s}{r_2(1 - s)^n}$$

$$\frac{dT_m}{ds} = -\frac{E^2 sn}{r_2(1 - s)} \tag{29a}$$

where $E$ is the voltage across the secondary load resistance $r_2/s$.

However, the electrical torque,

$$T_e = \frac{E^2 s}{r_2} \tag{30a}$$

and

$$dT_e = \frac{E^2}{r_2} ds + \frac{2Es}{r_2} dE \tag{31a}$$

Solving 29$a$ for $ds$ and substituting in eq 31$a$

$$dT_e = -\frac{(1 - s)}{sn} dT_m + \frac{2Es}{r_2} dE \tag{32a}$$

$dT_m = dT_e$ under steady state conditions, therefore,

$$\frac{dT_e}{dE} = \frac{2s^2 nE}{r_2(sn - s + 1)} \tag{33a}$$

or in terms of network power and voltage,

$$\frac{dP_a}{dE_a} = -\frac{2s^2 nE_a}{r_2(sn - s + 1)} \tag{34a}$$

Where $E_a = E$, the voltage across the secondary resistance $r_2/s$ and $P_a$ is the real power at point ($a$) into the network.

When $n = 0$, constant shaft torque load,

$$\frac{dP_a}{dE_a} = 0$$

## References


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# News

## Of Institute and Related Activities

## New Unified Publication Plan Facilitates Timely Release of Technical Papers

THE PURPOSE of this article is to explain some of the details of procedure involved in the handling of technical papers and special manuscripts under the new unified and enlarged publication program sponsored by the publication committee and adopted by the Institute's board of directors August 8, 1933. The general features of the plan were outlined in the September issue, and the changes as they apply to ELECTRICAL ENGINEERING and the TRANSACTIONS were treated at some length in the October issue.

### TECHNICAL PAPERS

Of advantage to the membership and to authors alike is the opportunity provided for the month-by-month publication of technical papers quite irrespective of their possible subsequent presentation before a convention or District meeting session. Heretofore papers have found their way into publication, and thus into the hands of the membership, only by virtue of being definitely scheduled for presentation before one of the technical sessions. This arrangement, with formal presentation practically the only recognized channel leading to publication, naturally contributed to an undesirable crowding of meeting programs and also interfered seriously with a satisfactory publication program by causing papers to be periodically released for publication in large quantities, with nothing in between times. Now, technical papers can be published throughout the year promptly upon their completion and review, leaving the question of possible scheduling for presentation at convention sessions to be settled independently in accordance with the requirements of the type of program that may be desired for any session or group of sessions.

With this more flexible relationship established between the Institute's technical publications and its technical meetings, substantial improvements may be made in both activities. On the one hand, the Institute's technical program can now be planned, evolved, and executed with a view toward maximum effective service to the membership at large through the channel of the monthly publication. On the other hand, the advance publication of all papers, and their wide circulation throughout the membership, paves the way for an improvement in the character and value of the technical sessions through the improved opportunity that is offered for the preparation of discussion, and through the additional time available at the sessions for the presentation of such discussion.

With respect to manuscripts, all contributions will be gratefully received by the Institute and will be reviewed thoughtfully to determine their publication possibilities. This applies to the more formal papers that ultimately may be scheduled for presentation at some convention or District meeting session, as well as to special articles such as those that have proved to be of such wide general interest in ELECTRICAL ENGINEERING since they were introduced in January 1931. Established practice will prevail in the handling of manuscripts.

The formal technical papers will continue to come largely from or through the Institute's 18 technical committees; through these committees the membership thus will be kept in touch with the latest in technological and related developments. As in the past, these papers undoubtedly will continue to originate through A.I.E.E. Section or Branch presentations, through research and other activities of educational institutions and industrial organizations, and through the various activities to which attention is given by the technical committees.

Recognizing the expressed demand of the membership for a continuation of the publication committee's policy made effective in January 1931 after an exhaustive study, special manuscripts of suitable character will continue to be drawn from any available authoritative sources. During the past 3 years these have included special presentations before Section or Branch meetings, papers or technical reports of other societies, and articles specially prepared for ELECTRICAL ENGINEERING by eminent authorities in various fields. In this connection in particular, the active coöperation of Section and Branch officers and other interested parties is solicited to the end that the publication committee and the editorial staff may be kept informed of possible sources of material of general interest and value to the membership.

It is of particular importance to note that, while the Institute solicits the submission of suitable manuscripts dealing with subjects of interest and importance to the electrical engineering profession, the acceptance of any manuscript for review and consideration does not constitute a commitment for its publication. The character and value of a manuscript cannot be determined in advance of its actual review. Also, it should be understood that acceptance of a paper for publication does not imply its inclusion in any given issue. To maintain a satisfactory quality and variety of subject matter in any given issue, it is necessary to select for that issue the best available material that will provide a desirable subject balance.

A further announcement, setting forth some of the details incident to the handling of technical discussion under the unified publication program, will be included in a subsequent issue of ELECTRICAL ENGINEERING.

### RELATION OF THE INSTITUTE TO ITS PUBLICATIONS

Another matter of importance involves the question of the degree of responsibility assumed by the Institute for statements made in its published technical material. For various reasons, some have chosen to regard the publication of a manuscript by the Institute as constituting sponsorship or endorsement by the Institute of the statements or inferences included in the manuscript published. This interpretation of the situation is unqualifiedly in error. The only published matter issued by the Institute that can be said to be recommended, endorsed, or sponsored by the Institute, are such publications as the various A.I.E.E. standards or other similar material issued in accordance with the constitution and by-laws and by authority duly conferred by the board of directors; on all such material there appears a definite statement indicating the extent to which such material is the official expression of the Institute as a body.

In issuing its technical publications, the Institute is merely discharging part of its charter obligation to undertake responsibility for the "advancement of the theory and practice of electrical engineering and of the allied arts and sciences, and of the maintenance of high professional standing among its members." In pursuit of this objective, and looking toward the edification of its membership, it becomes the obvious responsibility of the Institute to search out from all possible authoritative sources timely information of technical or professional importance including expressions of opinion and experience from recognized authorities, and to place this material in the hands of its membership. True, the acceptance and publication of a manuscript by the Institute constitutes a justifiably prized recognition of merit in a contribution to technical and professional literature. This recognition, however, by no means constitutes endorsement.

In its unceasing effort to discharge its publication obligation in a creditable and constructive manner, the Institute will continue to scrutinize critically all manuscripts submitted, and the technical qualifications of the authors that submit them, so that the membership may expect to receive the highest quality material available.

—Publication Committee

# A.I.E.E. Directors Meet at Institute Headquarters on October 20

THE regular meeting of the board of directors of the American Institute of Electrical Engineers was held at Institute headquarters, New York, N. Y., October 20, 1933.

Present were: *President*—John B. Whitehead, Baltimore, Md. *Past-presidents*—H. P. Charlesworth, New York, N. Y.; and C. E. Skinner, Wilkinsburg, Pa. *Vice-presidents*—K. A. Auty, Chicago, Ill.; A. H. Hull, Toronto, Ont.; J. Allen Johnson, Buffalo, N. Y.; E. B. Meyer, Newark, N. J.; and A. M. Wilson, Cincinnati, Ohio. *Directors*—L. W. Chubb, East Pittsburgh, Pa.; A. B. Cooper, Toronto, Ont.; P. B. Juhnke, Chicago, Ill.; A. E. Knowlton, New York, N. Y.; G. A. Kositzky, Cleveland, Ohio; Everett S. Lee, Schenectady, N. Y.; A. H. Lovell, Ann Arbor, Mich.; L. W. W. Morrow, New York, N. Y.; A. C. Stevens, Schenectady, N. Y.; and R. H. Tapscott, New York, N. Y. *National treasurer*—W. I. Slichter, New York, N. Y. *National secretary*—H. H. Henline, New York, N. Y.

Minutes of the directors' meeting of August 8, 1933, were approved.

A minute, published elsewhere in this issue, was adopted in memory of the late Dr. Farley Osgood, a past-president of the Institute.

A report was presented and approved of a meeting of the board of examiners held September 28, 1933. Upon the recommendation of the board of examiners, the following actions were taken upon pending applications: 1 applicant was transferred to the grade of Fellow; 15 applicants were transferred to the grade of Member; 4 applicants were elected to the grade of Member and 42 were elected to the grade of Associate as of November 1, 1933; 290 Students were enrolled.

Authority was given for the organization of a Student Branch of the Institute at Villanova College, Villanova, Pa.

The by-laws of the Institute were amended as follows:

The name of the committee previously known as the public policy committee changed to "Institute policy committee" wherever it occurs in the by-laws.

Secs. 51, 53, 57, and 58 revised to read as follows, as the result of recommendations adopted at the conference of Student Branch counselors held during the summer convention in Chicago, last June:

Sec. 51. Any person registered as a student, graduate, or undergraduate, in a university or technical school of recognized standing, who is pursuing a regular course in preparation for the profession of electrical engineering, may be enrolled as a Student of the American Institute of Electrical Engineers, except that a graduate student whose term of enrolment, as specified in Section 53, has expired may not again enroll. The expression "university or technical school of recognized standing" is interpreted as applying to any school of college grade which provides an engineering curriculum and is considered by the committee on Student Branches to maintain suitable qualifications in entrance requirements, content of curriculum, proficiency of teaching staff, and laboratory equipment.

Sec. 53. The annual Student enrolment fee shall be three dollars ($3.00) payable in advance and shall cover the fiscal year of the Institute beginning on May first. The initial payment upon application for enrolment shall be on the basis of a full year's fee, three dollars ($3.00), or a half year's fee, one dollar and fifty cents ($1.50), depending upon whether the application is filed nearer to May first or November first, respectively. The term of Student enrolment shall not extend beyond the end of the fiscal year in which student status ceases, nor for a period of more than five consecutive years for those students who are not devoting their entire time or a major part of their time to studies.

Sec. 57. Students registered in engineering subjects in schools other than those described above, while not qualified for Student enrolment in the Institute, may nevertheless subscribe for ELECTRICAL ENGINEERING at reduced rates by applying to the national secretary. The applications for such subscriptions must be accompanied by a certification from the instructor in charge of the engineering classes or department that the applicant is a student in engineering subjects at that school. These subscriptions are for a term of one year only, but may be renewed twice upon similar certification.

Sec. 58. Any member of the Institute who is a member of the electrical engineering faculty of a university or technical school of recognized standing may, upon the approval of the board of directors, organize a Student Branch under the designation (name of institution) Branch of the American Institute of Electrical Engineers.

The Branch activities in a school of recognized standing offering a curriculum in electrical engineering for evening students may, upon the approval of the committee on Student Branches, be conducted in two divisions to be known as "day" and "evening" divisions, each having officers chosen entirely from the corresponding group of students.

The board of directors may at any time terminate the existence of any Student Branch when in its judgment the interests of the Institute make such action desirable.

Upon the recommendation of the publication committee, the following subscription rates for Institute publications were adopted, effective January 1, 1934: ELECTRICAL ENGINEERING—non-member subscription price, $12.00 per annum, with extra postage charge for subscriptions from countries to which the bulk rate of postage does not apply; discount of 25 per cent to college and public libraries; discount of 15 per cent to publishers and subscription agencies. TRANSACTIONS—available to members upon advance subscription at a price not to exceed $5.00 per year; non-member subscription price, $12.00 plus extra foreign postage; 25 per cent discount to college and public reference libraries, and 15 per cent discount to publishers and subscription agencies.

Previous action of the board of directors, in prescribing a registration fee for non-members at national conventions and District meetings of the Institute, was modified to exempt Enrolled Students of the Institute and the wives and children of members.

The North Eastern district (No. 1) of the Institute, upon request of its executive committee, was authorized to hold a District meeting, in Worcester, Mass., in the spring of 1934, with the understanding that it will involve no expenses chargeable to the national treasury.

Certain suggestions relating to the celebration, in May 1934, of the fiftieth anniversary of the Institute, were considered and adopted.

As required by the by-laws, 5 members of the board of directors were selected to serve as members of the national nominating committee, as follows: H. P. Charlesworth, G. A. Kositzky, L. W. W. Morrow, A. C. Stevens, and H. R. Woodrow.

The following actions were taken upon the recommendation of the standards committee:

Approved report on standards for acceptance tests for metal tank mercury arc rectifiers, prepared by the sectional committee on mercury arc rectifiers, with the understanding that the Institute is prepared to print it either as a report or as a tentative standard as may be decided by the American Standards Association;

Authorized the Bureau of Standards to publish under one cover all letter symbols published by the Institute and which have been approved as American Standard;

Authorized a request to the American Standards Association for the extension of the scope of the sectional committee on railway motors to include all rotating electrical machinery for use in connection with the power equipment of electrically propelled cars or locomotives.

The finance committee reported monthly disbursements amounting to $15,687.97 for September and $24,007.66 for October.

A budget for the appropriation year beginning October 1, 1933, submitted by the finance committee, was adopted.

Representatives were appointed as follows:

H. R. Woodrow was appointed a representative of the Institute on United Engineering Trustees, Inc., for the 3-year term beginning in January 1934, succeeding H. A. Kidder, whose term will expire at that time and who is ineligible for reappointment;

Professor W. I. Slichter was nominated for reappointment to the Engineering Foundation board for the 3-year term beginning in February 1934;

R. N. Conwell was appointed as a representative of the committee on power transmission and distribution on the American committee on marking of obstructions to air navigation.

Upon invitation from the Stevens Institute of Technology to coöperate in a celebration, on December 7, 1933, of the fiftieth anniversary of the graduation of Frederick Winslow Taylor, the president and the secretary were empowered to prepare a suitable tribute for this occasion, and the president was authorized to appoint delegates to attend the meeting.

Resolutions were adopted, establishing policies for the ensuing year in conformity with the budget adopted by the board, covering a fiftieth anniversary issue of ELECTRICAL ENGINEERING, May 1934;

## Future AIEE Meetings

**Winter convention,**
New York, N. Y., Jan. 23–26, 1934

**North Eastern District meeting,**
Worcester, Mass., Spring 1934

**Summer convention,**
Hot Springs, Va., June 25–29, 1934

**Pacific Coast convention,**
Salt Lake City, Utah, Sept., 1934

the omission of all cash prizes for papers presented during the calendar year 1933, with the exception of the District prizes for Branch papers, but the presentation of certificates as usual; a decrease in the number of meetings of the board of directors, with the substitution of meetings of the executive committee at the discretion of the president; a request to the Sections for coöperation in reducing Institute expenditures by keeping the total for each Section at least 20 per cent below the maximum amount available on the basis given in the by-laws; and listing all traveling expenses chargeable to the national treasury.

Other matters were discussed, reference to which may be found in this and future issues of ELECTRICAL ENGINEERING.

## November 15 Last Date for Suggesting Nominations

The national nominating committee of the A.I.E.E. will meet between November 15 and December 15, 1933, for the purpose of nominating Institute national officers to be voted upon in the spring of 1934. The articles from the constitution and by-laws regarding the procedure of the national nominating committee and the method of submitting suggestions to it were contained in ELECTRICAL ENGINEERING for October 1933, p. 720; this article also called attention to the fact that members are invited to suggest nominations up to November 15, 1933. For those members who may not have seen the previous article, this invitation is repeated here. To be available for the consideration of the committee all such suggestions must be received by the secretary of the committee at Institute headquarters, New York, N. Y., not later than November 15, 1933.

### INDEPENDENT NOMINATIONS

The nominations as made by the national nominating committee are required by the by-laws to be published in the January issue of ELECTRICAL ENGINEERING or otherwise mailed to the Institute's membership during the month of January. Attention of the membership is hereby called to the fact that additional nominations may be made independent of those of the national nominating committee as late as February 15. The following provisions quoted from the constitution and by-laws govern such independent nominations:

**Constitution**

Sec. 31. Independent nominations may be made by a petition of 25 or more members sent to the national secretary when and as provided in the by-laws; such petitions for the nomination of vice-presidents shall be signed only by members within the district concerned.

**By-laws**

Sec. 23. Petitions proposing the names of candidates as independent nominations for the various offices to be filled at the ensuing election, in accordance with Article VI, section 31 of the constitution must be received by the secretary of the national nominating committee not later than February 15 of each year, to be placed before that committee for the inclusion in the ballot of such candidates as are eligible.

On the ballot prepared by the national nominating committee in accordance with Article VI of the constitution and sent by the national secretary to all qualified voters during the first week in March of each year, the names of the candidates shall be grouped alphabetically under the name of the office for which each is a candidate.

## Notice of Change in Non-Members' Subscription Rates

Effective January 1, 1934, the A.I.E.E. monthly publication, ELECTRICAL ENGINEERING, will be enlarged materially, as already has been announced, and the A.I.E.E. TRANSACTIONS will be changed from a quarterly to an annual cloth bound volume to be issued each December embracing the entire content of the 12 monthly issues. This enlarged service requires a nominal increase of rates to non-members of the Institute.

Therefore, pursuant to the actions of the board of directors of the American Institute of Electrical Engineers, as recorded elsewhere in this and preceding issues of ELECTRICAL ENGINEERING, notice is hereby given that, effective January 1, 1934, subscription rates to ELECTRICAL ENGINEERING and to the A.I.E.E. annual TRANSACTIONS for non-members of the Institute will prevail as outlined in the following paragraphs.

**To Individual Non-Members.** ELECTRICAL ENGINEERING annual subscription: in the United States and possessions, and elsewhere where U.S. bulk mailing rates prevail, $12; Canada and Mexico, $13, including $1 extra postage; all other points, $14, including $2 extra postage. All subscriptions payable in advance in New York exchange.

A.I.E.E. TRANSACTIONS in annual bound volume form to be issued in December of each year beginning 1934: in the United States and possessions, and elsewhere where U.S. bulk mailing rates prevail, $12; Canada and Mexico, $13, including $1 extra postage; all other points, $14, including $2 extra postage. All subscriptions payable in advance in New York exchange.

Combination subscriptions may be made to the advantage of the subscriber. If both the monthly publication, ELECTRICAL ENGINEERING, and the annual bound volume of TRANSACTIONS are subscribed for at one time, and the subscription rates as above quoted paid in advance, the double publication service may be obtained at the annual rate of $16 (New York exchange), plus foreign postage as stated above.

**Colleges and Public Libraries.** Established libraries of recognized educational institutions, and bona fide public reference libraries may subscribe to ELECTRICAL ENGINEERING or to the annual TRANSACTIONS at the single or combination rates stipulated for individual non-members of the Institute, less 25 per cent discount applied to the base rate (discount not applicable to extra postage charges).

**Subscription Agencies.** Recognized subscription agencies that have established a satisfactory business relationship with Institute headquarters will be allowed a 15 per cent net discount as applied to the base subscription rates quoted for individual non-members of the Institute (discount not applicable to extra postage charges).

**Single Copies.** For single copies of the above mentioned publications, when available, prices will be quoted upon application.

(*Signed*) H. H. HENLINE,
*National Secretary*

October 20, 1933.

## 4-Day Winter Convention Scheduled for 1934

The annual winter convention of the A.I.E.E. held in the Engineering Societies Building, New York, N. Y., each year, will differ from other recent winter conventions in that it will start on a Tuesday, one day later in the week than before. The dates of the convention have been established as January 23–26, 1934. Technical sessions will be held on Tuesday, Wednesday, and Thursday, Friday being left open for inspection trips and other special features. Plans for the convention are progressing rapidly, the general committee having been appointed, 12 technical sessions arranged, and other events of technical and social interest scheduled. Detailed plans will be announced in future issues of ELECTRICAL ENGINEERING.

Three symposiums are planned, one on electric furnaces, another on electric power distribution, and a third on electric power transmission. A session on recent improvements in sound transmission and reproduction in auditory prospective also has been scheduled. Other sessions are being arranged on the following subjects: electrical measurements, transportation, protective devices, power generation, electrical machinery (2 sessions), electric welding, and automatic stations.

Another feature of this coming winter convention will result from the Institute's new publication plan, whereby, in so far as possible, papers to be presented at the winter convention will have been published in ELECTRICAL ENGINEERING previous to the date of the convention, thus making possible a careful advance study of these papers.

### GENERAL COMMITTEE APPOINTED

The special convention committees, so important to a successful carrying out of the program, now are being formed. Announcement was made on October 10 of the appointment by President Whitehead of the following general committee in charge of arrangements for the 1934 winter convention: *Chairman*, C. R. Jones (A'16, M'30) secretary District 3 and chairman New York Section; T. F. Barton (A'12, F'30) member, special committee on model registration law; C. R. Beardsley (A'08, F'30) secretary, New York Section; C. O. Bickelhaupt (M'22, F'28) Institute representative on Engineers Council for Professional Development and other bodies; R. N. Conwell (A'15, F'31) chairman, technical program committee; A. F. Dixon (A'14, F'26); E. B. Meyer (A'05, F'27), vice-president District 3; H. S. Osborne (A'10, F'21), chairman committee on communication; and D. M. Simmons (A'22, F'28), chairman committee on power transmission and distribution.

## Engineering Education Commented Upon by Dean Sackett

OBSERVATIONS on engineering education, especially as pertaining to the broader social point of view required of engineering teachers in a world where engineering plays an ever-increasing part, are contained in a few brief notes prepared by R. L. Sackett, dean of engineering of The Pennsylvania State College, and chairman of the committee on student selection and guidance of the Engineers Council for Professional Development. These notes were prepared for ELECTRICAL ENGINEERING by Dean Sackett, at the request of L. A. Doggett (A'13, M'16) professor of electrical engineering, The Pennsylvania State College, and chairman of the Institute's committees on Student branches and on education for the year 1933–34. Dean Sackett's notes follow:

"At first sight nothing seems to be securely anchored in this changing world. Engineering education is not immutable but neither is it drifting. It is assailed as a part of the complex cause of our inevitable march toward lessened physical burdens and increased social responsibility. We are not inclined to shirk our responsibility for contributing our part toward the construction of a science minded world nor are we proposing to evade our duty.

"Those who have financed and directed this engineering world have misunderstood its purpose, which is to bring comfort, safety, health, and a larger measure of satisfaction to men. Wealth which has grown as a result of engineering application is a means—not an end.

"No one, who has read the 1923–29 report of the investigation of engineering education prepared by the Society for the Promotion of Engineering Education, doubts that engineering education has been technically effective—too effective, because the rest of society has not kept step with the engineer. Judging from the recent past and the present, the producer, the consumer, and capital are out of step. The lawyer, the financier, and the economist have so far done little toward synchronizing the social machinery. It seems to be in part the job of the engineer to transform materials and forces of nature and the labors of men that they may contribute to the benefit of mankind—to paraphrase Telford's definition of engineering.

"The engineering teacher must get in phase rather than leave coördination so largely to others. We were guilty of emphasizing the materials and forces of nature but have made amends by more recently giving due weight to the importance of management and its criterions. Now we must emphasize to our teachers and students the lessons of adversity; *viz.*, that to economy and efficiency in design, construction, and operation there must be added a recognition of social liability for invention and efficiency.

"Those who have studied the development of engineering during these hundred years with its increasing complexity, its broadening scope and the scientification of its procedure cannot but be disturbed by the obvious need for the engineer to sharpen his knowledge, remagnetize his experience and orient his direction finder so that he may join other sensitized forces in energizing sound industrial and social aims to the end that engineering may serve more equitably and uniformly than it has.

"In education, it seems to me that the implications are obvious. Until there is a better program, it devolves on the engineering teacher to formulate his philosophy of industry and engineering and to present it as an integral doctrine so that where engineering reacts on society—and that is everywhere—the social effects may be understood. Matured observation and study kneaded into the lesson and leavening engineering teaching is a step forward which can be taken in humanizing technology.

"New courses of study may be developed to meet the need but in the meantime a change of emphasis by the more mature and alert engineering teachers is desirable in order that the graduate may be conscious of the widening influence of engineering and his obligations as a technician and as a citizen."

## Visitors to Chicago

GUGLIELMO MARCONI (HM'17) president of the Marconi Wireless Telephone Company, London, England, and Dr. E. F. W. ALEXANDERSON (A'04, F'20) consulting engineer, General Electric Company, Schenectady, N. Y., photographed together in Chicago at the 1933 World's Fair. The occasion was "Marconi Day" at the fair, during which Senatore Marconi left, was the honored visitor.

**Doctor Fink to Receive Perkin Medal.** Dr. Colin G. Fink, professor of electrochemistry at Columbia University, has been elected to receive the Perkin Medal of the Society of Chemical Industries for 1934. The medal is awarded annually for work in applied chemistry and will be presented this year to Doctor Fink for his inventions of the fields of metallurgy and electrochemistry. The selection is made by a joint committee of members of the Society of Chemical Industries, the American Chemical Society, the American Institute of Chemical Engineers, the Electrochemical Society, and the Société de Chimie Industrielle. Presentation of the medal will be made at a meeting in New York, N. Y., early in January, 1934.

## Boulder Dam Generators Ordered

The 5 generators which will comprise the first unit for the development of electric power at Boulder Dam, and will make available for Los Angeles, Calif., and vicinity, a total of 300,000 kw, have been placed on order with 3 different manufacturers. Contracts for these units have been awarded by the United States Bureau of Reclamation of the Department of the Interior. Four units, each rated 82,500 kva, will be the largest water wheel generators in the United States, and in capacity will exceed any other generators now in operation. All 5 generators are of the vertical shaft type.

Two of the generators, each rated at 82,500 kva, unity power factor, 3 phase, and designed for 50 cycle generation at 150 rpm and 13,800 volts, or 60 cycle generation at 180 rpm and 16,500 volts, have been ordered from the General Electric Company. Work is now being started on the construction of these units at the Schenectady, N. Y., plant of the company, with the first unit scheduled for completed installation early in 1935, and the other unit later that year. Two other 82,500-kva generators have been ordered from the Westinghouse Electric and Manufacturing Company, the first of these generators to be delivered in about $1\frac{1}{2}$ years. Work on the units will start soon at the East Pittsburgh, Pa., works. The fifth unit, rated 40,000 kva, will be built by the Allis-Chalmers Manufacturing Company, at West Allis, Wis.; this is a unity power factor 13,800 volt, 3-phase, 60-cycle generator to operate at 257 rpm. The machine is equipped with a main and a pilot exciter; the outside diameter of its yoke will be approximately 32 ft, and the shipping weight of the total generator approximately 400 tons. The machine is to be completed in 515 days after the government has approved its drawings. Ultimate plans at the Boulder Dam station call for 15 of the 82,500-kva generators and 2 of the 40,000-kva generators.

Announcement is made by the Westinghouse company that approximately 600,000-man-hours of work will be required to build the 2 units awarded to this company. The General Electric Company announces that the units awarded to it will be smaller in diameter but both heavier and higher than generators recently built for the U.S.S.R.; these Boulder Dam generators will have an overall diameter of 40 ft, and a height of 32 ft above floor level. Each will weigh more than 1,000 tons. The rotor, including

its 57.5-ton 38-in. diam. shaft, will weigh 625 tons. At least 40 freight cars will be required for the transportation of each unit, including generator, regulator, surface air coolers, and exciters, from Schenectady to Boulder Dam. The frame will be shipped in four sections. Because of special problems encountered in the transmission of the power over the distance of 265 miles to Los Angeles, special features of design will be incorporated in the units, making them equivalent in size to generators of approximately 125,000-kva capacity in units of normal characteristics. The water-wheels driving the generators will operate 90 per cent of the time with a head of from 450 to 560 ft, with minimum and maximum limits of 420 and 590 ft.

The Boulder power house, at the Boulder Dam on the Colorado River, approximately 25 miles southeast of Las Vegas, Nev., will be immediately downstream from the dam. It will be a U-shaped structure, with the base of the U across the river on the down stream toe of the dam, and one wing on each side of the river. The main generating units and the station service units will be in the wings of the power plant, and the central portion will contain the sump pumps, machine shop, control, and auxiliary equipment. The main step-up transformers and the low voltage switching equipment will be on a platform along the river side of the wings of the power house. The high voltage switching station will be on top of the canyon, a short distance back from the rim on the Nevada side of the river. Overhead high voltage circuits will connect each bank of step-up transformers with the switching station.

Two of the 82,500-kva generators normally will be operated in parallel and will be connected to one bank of step-up transformers, consisting of 3 55,000-kva single-phase transformers connected delta on the low voltage side and star on the high voltage side. A 13,800/16,500-volt transfer bus will be provided so that a spare 82,500-kva generator may be substituted for any other 82,500-kva generator without dropping load, and the low voltage switching equipment will have sufficient interrupting capacity to permit 3 main generators being connected to the transfer bus simultaneously. The power generated by these units will be transmitted to the city of Los Angeles by means of 2 transmission circuits operating at 275,000 volts at the receiving end and from 280,000 to 303,000 volts at the sending end. The 265-mile transmission lines will be sectionalized into 3 approximately equal sections by intermediate switching stations.

**Chemical Engineers Change Meeting Date.** The American Institute of Chemical Engineers has changed the date of its national meeting to the week following that originally scheduled. The meeting now will be held December 12–14, 1933; the place of the meeting is unchanged, namely, Roanoke, Va. The purpose of this change in date is to make it possible for individuals to attend the Exposition of Chemical Industries to be held in New York, N. Y., December 4–9, 1933, as well as the sessions of the A.I.C.E. in Roanoke.

## In Memoriam

FARLEY OSGOOD

ON THE sixth of October there passed from the ranks of the Institute another of its loyal workers. When death took from us, Dr. Farley Osgood, past-president, we lost a man of unusual attainments, a friend always to be depended upon. Fearless and outspoken, he brought to bear on the many tasks that came before him in life a vigorous personality and an understanding of human nature that guided him swiftly and surely in all his dealings.

Entering the Institute as an Associate in 1905 during the period when he was affiliated with the New York and New Jersey Telephone Company, he soon became a leader in A.I.E.E. development, attaining Fellowship in 1912. Serving on the board of directors as director and vice-president from 1911 to 1916, Farley Osgood at the same time took an active part in the work of many A.I.E.E. committees, giving freely of his services to the standards committee, the finance committee, the Edison Medal committee, and to many of the joint co-operative undertakings of the engineering field. Transferring his business connections from the telephone to the power field, he eventually became vice-president and general manager of the Public Service Electric and Gas Company of New Jersey, and it was during his incumbency of that office that he was elected President of the Institute, 1924–1925.

It is, therefore, with a keen appreciation of the part which Farley Osgood played in A.I.E.E. history that the board of directors hereby expresses its deep sorrow at his death and resolves that this minute be spread upon the Institute records and be transmitted to his family and associates.

**A.S.C.E. Nominates Officers for 1934.** The "official nominees" for positions as officers of the American Society of Civil Engineers for the term beginning January 17, 1934, have been elected. For president, Harrison P. Eddy, consulting engineer, Boston, Mass., was nominated. Two vice-presidents were nominated, J. P. Hogan (M'31) consulting engineer, New York, N. Y., to represent zone I, and H. D. Dewell, consulting engineer, San Francisco, Calif., to represent zone IV. Five directors were nominated; these directors and the districts which they represent are: district 1, O. H. Ammann, chief engineer, Port of New York Authority, New York, N. Y., and C. E. Trout, consulting engineer, New York, N. Y.; district 2, F. A. Barbour, consulting engineer, Boston, Mass.; district 6, T. J. Wilkerson, county engineer, Beaver Falls, Pa.; and district 10, F. H. McDonald, consulting engineer, Atlanta, Ga. In district 13, there was a tie vote in the nomination for director, and the election to determine the director from this district will be held in January; the 2 men to be voted upon are T. E. Stanton, Jr., materials and research engineer, State Department of Public Works, Sacramento, Calif., and B. A. Etcheverry, professor of irrigation and drainage, University of California, Berkeley.

## E.C.P.D. Elects Officers

At a meeting of the Engineers Council for Professional Development held October 10, 1933, officers for the coming year were elected. C. F. Hirshfeld (A'05), chairman *pro tem*, was elected chairman, and C. E. Davies, secretary *pro tem*, was elected secretary. The group which had been serving as an interim executive committee also was elected as an executive committee for the coming year. This committee includes: J. V. Davies, W. E. Wickenden (A'07, M'13), C. F. Scott (A'92, F'25, HM'29), H. C. Parmelee, D. F. Irvin, R. I. Rees, D. B. Steinman, with C. F. Hirshfeld and C. E. Davies, *ex-officio* members.

A news item outlining the organization of the E.C.P.D., and including the initial program to be followed, was given in ELECTRICAL ENGINEERING for October 1933, p. 722. As stated in that item, 4 working committees have been appointed and each have reported progress in its particular field.

**Fifth Unit at Safe Harbor Placed in Operation.** An additional 42,500-hp generating unit was placed in operation at the Safe Harbor hydroelectric development on the Susquehanna River, during the last week in September. Emphasis is placed by officials of the Safe Harbor Water Power Corporation and the Pennsylvania Water and Power Company on the fact that this plant, which was started during the depression, has contributed substantially to employment, the work carried on during the summer and fall of 1933 having required the services of some 400 men; this figure includes various related construction jobs in

addition to the installation of the fifth unit. During the years 1930–32, expenditures of upwards of $28,000,000 were made by these 2 hydroelectric companies in the power house and extensions of the transmission system; of this amount over $9,000,000 was spent for labor and engineering. A description of the Kaplan turbines at Safe Harbor and a general description of the development are given in ELECTRICAL ENGINEERING for October 1933, p. 728–33, and November 1932, p. 757–65.

**A.S.M.E. to Hold Annual Meeting.** The 54th annual meeting of The American Society of Mechanical Engineers will be held in the Engineering Societies Building, 29 West 39th Street, New York, N. Y., Dec. 4–8, 1933. Some 35 sessions have been arranged for this meeting, which include many papers of interest to electrical engineers. A few of these subjects are electric furnaces, air conditioning, power generating stations, industrial power, light weight railway equipment, economics, and education.

**Unequal Price Changes Shown By Report.** The difficulty of meeting past obligations at present price levels, a difficulty that is fully recognized, has been so strongly emphasized that it has obscured the disturbance caused by the unequal fall of different prices, says the National Industrial Conference Boards New York N. Y., in a bulletin on price changes since 1929. Business difficulties arise not merely from a fall in general prices, but from variations in the fall of prices of different commodities or groups of commodities. The conference board points out that there are prices for commodities at wholesale, for commodities at retail, and prices for services, that these classes have not fallen in the same measure, and that within each class the component elements show divergent degrees of price decline. Although the United States bureau of labor statistics index of wholesale prices shows a drop of 36 per cent from 1929 to January, 1933, wide deviations from the average are revealed when the individual commodities that compose this index are examined separately or by groups and subgroups. By groups, the price change in January 1933, varied from a drop of 59.4 per cent for farm products to a drop of only 20.5 per cent for fuel and lighting materials. By subgroups, the movements ranged from a drop of 84.6 per cent for crude rubber to a rise of 9.2 per cent for electricity. By individual commodities, the movements ranged from a decline of 85.8 per cent in crude rubber, amber, No. 3, to a rise of 77.9 per cent in the price of hops. The conference board has computed for 715 comparable price quotations the percentage changes from the 1929 averages to the prices of January 1933. While retail prices are not so fully recorded as wholesale prices, the retail prices that are included in the conference board's cost of living index show generally less decline than the nearest corresponding groups in the wholesale price index. The cost of living as a whole has declined less than retail prices and much less than wholesale prices, because a considerable number of the services included in the cost of living have changed but little.

**Naval Architects and Marine Engineers to Meet.** The 41st annual meeting of the Society of Naval Architects and Marine Engineers will be held in the Engineering Societies Building, 29 West 39th Street, New York, N. Y., Nov. 16–17, 1933. Twelve technical papers have been scheduled.

## A Substation on the 132-Kv British "Grid"

NORTHFLEET switching and transforming station, shown here, is stated by The Travel and Industrial Development Association of Great Britain and Ireland to be the most powerful load dispatching center in the national "grid" system of Great Britain. From it 7 circuits, each having a capacity of 50,000 kw at 132 kv, radiate to South East England and East England.

**Employment Service Active in San Francisco.** Reports from the San Francisco, Calif., office of the Engineering Societies Employment Service received from F. R. GEORGE (A'13, M'25) representative of the Institute's San Francisco Section on the advisory council of this office, have been very encouraging this year. During the first 9 months of 1933 more placements have been made by the San Francisco office than during the entire year in either 1931 or 1932, and the outlook is for a better record than in 1930. The San Francisco office is one of the 3 offices of the Engineering Societies Employment Service, maintained by the national societies of civil, mining, mechanical, and electrical engineers, in coöperation with the Western Society of Engineers, Chicago, and the Engineers' Club of San Francisco.

## A Piezo Oscillator of 200 Kc Per Second

A piezo oscillator which is used to control the frequency of the standard frequency transmitter of the U.S. Bureau of Standards is described in research paper No. 576, which appears in the July 1933 number of the Bureau of Standards Journal of Research. The piezoelectric element is a Curie or zero-cut quartz plate having a fundamental frequency of 200 kc per second. The mounting is a clamped type which maintains the quartz plate in a fixed position between the electrodes without introducing an excessive amount of damping.

The oscillator circuit arrangement is of the conventional type in which the quartz plate is connected between the grid and filament of a *UX112A* tube. An untuned inductance load is connected in the plate circuit. There are 2 coupling amplifiers which are connected in cascade. The first is a *UX222* tube which is connected to the oscillator plate impedance through a small condenser and a voltage divider arrangement. The second amplifier is a *UX112A* tube.

The temperature control consists of 2 thermostatically controlled compartments, one within the other. The quartz plate has a double temperature control while the oscillator and amplifier circuit arrangements are within the outer control only. The piezo oscillator described has been in use for a year. During this time its frequency in terms of the primary frequency standard has changed less than 1.5 parts in $10^6$. The frequency is constant within a few parts in $10^8$ for several hours.

**Chemical Industries to Hold Exhibit.** The 14th exposition of chemical industries will be held in Grand Central Palace, New York, N. Y., Dec. 4–9, 1933. These expositions are being held every second year, the exhibits reviewing the industrial progress in the chemical industries during the preceding 2 years. This exhibit will be held during the same week as the annual meeting of The American Society of Mechanical Engineers, also in New York City.

# Letters to the Editor

CONTRIBUTIONS to these columns are invited from Institute members and subscribers. They should be concise and may deal with technical papers, articles published in previous issues, or other subjects of some general interest and professional importance. ELECTRICAL ENGINEERING will endeavor to publish as many letters as possible, but of necessity reserves the right to publish them in whole or in part, or to reject them entirely.

STATEMENTS in these letters are expressly understood to be made by the writers; publication here in no wise constitutes endorsement or recognition by the American Institute of Electrical Engineers.

## Training of Engineers in the Power of Expression

*To the Editor:*

I have read with a great deal of interest various valuable contributions which have appeared from time to time regarding engineering education, and I have been much impressed with the emphasis upon the desirability of providing a thoroughly broad foundation in the technical school upon which to build, through experience, the specialization which necessarily comes later.

Although there is, I believe, a tendency among some engineering educators to overemphasize the policy of broad general training at the expense of certain definite engineering subjects (a policy which I fear may sadly handicap many young engineering school graduates) there is one general subject which I believe has not been given sufficient thought in our engineering schools —I refer to development of power of expression, both written and oral.

An engineering graduate may have been taught to think clearly on a number of various subjects in the engineering field, as well as in allied fields, but if he has not developed the ability to set clearly down on paper whatever thoughts he may have, and if he cannot arise before an audience and tell convincingly and clearly what he has in his mind, his usefulness to the community is relatively limited.

The engineering profession is suffering seriously from the fact that its practitioners are not articulate. It has been a matter of pride with many engineers that they can express themselves in *deeds* rather than in *words;* that they *do* things rather than talk about them. Of the 2 accomplishments there is no question which is the more valuable; nevertheless, the individual cannot attain maximum usefulness in the community unless he has both.

The young law school student knows that in order to practice his profession successfully he must train himself in power of expression; that if he cannot address an audience convincingly he will be very much handicapped, and if he cannot prepare a brief, arranged logically and expressed clearly, he will not be likely to succeed in his professional career. The modern law school therefore stresses training in public speaking and in writing, as a regular part of its curriculum.

The result of this is that lawyers have assumed the leadership in public affairs. This may or may not be for the best interest of the community. There is no reason to believe that lawyers have more ability or are better equipped as leaders than are engineers, and the inference is entirely logical that lawyers have been accepted as leaders very largely because they have developed powers of expression.

It is suggested, therefore, that the courses in engineering schools be expanded to include definite training in thought expression. This may be best accomplished undoubtedly through constant practice in public speaking. Such practice might well be carried on in connection with engineering courses, in the form of oral exposition of problems before classes, descriptive reports delivered to the faculty and students, and in debates; a surprising number of engineering subjects are controversial and thus lend themselves very well to debates.

If the engineer of the future is equipped upon graduation with the ability to express himself extemporaneously before an audience, and to state clearly and logically the things which he has in his mind, there is little question that the engineering profession will ultimately assume that leadership which it logically should have, in view of the accomplishments of engineers in adapting the laws of nature to the service of mankind.

Another thought which arises in this connection lies in the amount of time available normally for engineering training. Many educators complain bitterly that the standard 4-year course is too short for the effective training of engineers. In most cases during the "standard 4-year course" the student has 3 long summer vacations which might well be far more fully utilized in his engineering training than is the case in many schools today. There is no reason why the training should not be carried on continuously throughout the student's entire 4-year course. This would provide opportunity for the inclusion of many things which are not considered possible with present limitations which seem to be imposed by the calendar.

The advantages of making the engineering course a graduate course, analogous to courses in medicine, law, etc., are so obvious that there is no question that in the not distant future, competition in the engineering profession will bring about this desirable result. At that time the complaints expressed by teachers of engineers, of lack of time in which to accomplish all they would like to accomplish, will be very much less serious than they are today, in spite of the constantly increasing demands upon the young engineer.

Very truly yours,

SIDNEY WITHINGTON (M'20, F'24)
(Electrical Engineer, The New York, New Haven and Hartford RR. Co., New Haven, Conn.)

## A Graphical Device for Obtaining $\sqrt{a^2 + b^2}$

*To the Editor:*

I was much interested in the article by W. J. Seeley in ELECTRICAL ENGINEERING for August 1933, p. 583–4, on "Short Cuts to Finding $\sqrt{a^2 + b^2}$." As Mr. Seeley stated, many engineering calculations require the evaluation of the expression $\sqrt{a^2 + b^2}$ and sometimes a great amount of labor is expended in this solution. So much so that I found it almost necessary to construct what might be called a graphical slide rule, and shown in Fig. 1. It will solve, with slide rule accuracy, the equations $\sqrt{a^2 + b^2}$ or $\sqrt{a^2 - b^2}$ by only 2 settings. This does not take the place of the various methods brought out by Mr.

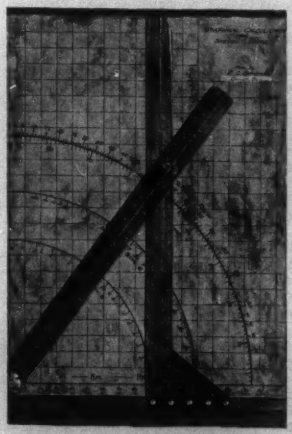

**Fig. 1. Graphical calculator**

Seeley where accuracy is necessary. However, for a large percentage of the practical engineering problems slide rule accuracy is sufficient and the rapid solution possible with the graphical calculator makes its value apparent.

As indicated in the illustration the T-square member carried a scale which would represent one side of the triangle, or reactive voltamperes, or reactance. The fixed horizontal scale would represent the other side of the triangle, or watts or resistance. The hinged member would represent the hypotenuse of the triangle, or voltamperes, or impedance. The 3 scales are identical. The solution of $\sqrt{a^2 + b^2}$ is obtained by setting the reference line of the T-square member to the value of $a$ on the fixed horizontal scale. The hinged member is moved until its reference line crosses that of the T-square member at the value of $b$. The answer is found on the hinged member at $b$.

This device has saved considerable time in the solution of electrical problems, including transmission line problems. Most any trigonometry problem involving a right-angle triangle can be solved without

reference to trigonometric tables. The sine, cosine, tangent, or cotangent of any angle can be read directly on the respective scales.

Yours very truly,
E. C. SCHURCH (A'24)
(Electrical Engineer, Box 284, Denver, Colo.)

## Accurate Roots by Calculating Machine

*To the Editor:*

The makers of calculating machines in general advise purchasers that square roots may be obtained by the subtraction of successive odd numbers, and that higher roots cannot be found at all. This successive subtraction entails much labor, and the chances for error are many.

The method here set forth is based on the fact that, in any rectangle, the *average* of the 2 adjacent sides is closer to the side of the equivalent square than is either original dimension. Given the area, we guess one side and by division find the other; then the average of the 2 may be considered the side of another rectangle, more nearly square than the first. By successively dividing and averaging, any desired degree of accuracy may be obtained.

Obviously, the number of steps required is an inverse function of the accuracy of the original guess: in general, one step will give accuracy sufficient for most engineering purposes when the first approximation is found by slide rule.

*Method a.* To find the square root of $A$, as $\sqrt{A} = a$: Obtain, by slide rule or otherwise, a first approximation, $a_1$. Then, by calculating machine find $A/a_1 = a_1'$ and the second approximation, $a_2 = (a_1 + a_1')/2$. By machine, again, $A/a_2 = a_2'$, and the third approximation, $a_3 = (a_2 + a_2')/2$. This process is continued until $(a_i - a_i')$ is less than the allowable error in $a$, when $a_i^2 = A$.

*Method b.* To find higher roots of $A$, as $\sqrt[n]{A} = a$. Obtain, as before, a first approximation, $a_1$. Then $A/a_1^{(n-1)} = a_1'$, and the second approximation, $a_2 = \frac{(n-1)a_1 + a_1'}{n}$. And so on until $(a_i - a_i')$ is less than the allowable error in $a$, when $a_i^n = A$.

*Accuracy of Method a.* The first approximation differs from the true value, $a$, by a factor $k_1$, so that $a_1 = k_1 a$; and $a_1' = a/k_1$ (since $a_1 a_1' = a^2$). Then $a_2 = (a_1 + a_1')/2 = a(k_1 + 1/k_1)/2 = k_2 a$; and $k_2 = \frac{1}{2}\left(k_1 + \frac{1}{k_1}\right)$.

That is to say, if the first approximation is in error by 1 per cent, $k_1 = 1.01$, and $k_2 = (1.01 + 0.990099)/2 = 1.00005$: The second approximation is accurate to one part in 20,000. One step, then, will give a sufficiently accurate result for most engineering purposes when a slide rule is used to find the first guess.

*Accuracy of Method b.* The advantage, naturally, is not as great for the higher roots. $a_1 = k_1 a$ and $a_1' = a/k_1^{(n-1)}$; $a_2 = [(n-1)k_1 + k_1^{(1-n)}]a/n = k_2 a$ so that $k_2 = \frac{(n-1)k_1^n + 1}{nk_1^{(n-1)}}$. Suppose $n = 5$ and $k_1 = 1.10$ (a 10 per cent error in the first guess). Then it is found that $k_2 = 1.01660$, and that $k_3 = 1.00053$; a satisfactory answer is secured on the second attempt. As is seen, the series converges with extreme rapidity.

*Examples*

1. Square Root. By slide rule, $\sqrt{5} = 2.235$, with the last figure in considerable doubt. On the machine, $5/2.235 = 2.237136465$, and the average of the 2 gives the second approximation, 2.23606823. This, squared, is 5.00000114; the error in $A$ is one part in 5 million, so that $a$ is accurate to one part in 10 million, whereas the first guess was good to one part in 2,000.

2. Cube Root. By slide rule, $\sqrt[3]{3} = 1.44$, and by machine, $3/(1.44)^3 = 1.446759259$. The second approximation, $(2 \times 1.44 + 1.446759259)/3 = 1.4422531$; cubed, is 3.0000219.

3. The fourth root, is of course, the square root of the square root.

4. Fifth root. By slide rule, $\sqrt{3} = 1.73$; $\sqrt{1.73} = \sqrt[4]{3} = 1.31$, while $\sqrt[4]{1.73} = \sqrt[5]{3} = 1.20$. The fifth root, then, lies between 1.20 and 1.31: choose, for the first guess, 1.25. Then $3/(1.25)^4 = 1.228800$, and the second approximation, $(4 \times 1.25 + 1.2288)/5 = 1.2457600$, raised to the fifth power, is 3.000350. By repeating the process, the third approximation is found to be 1.24573091, which, raised to the fifth, is 3.00000242.

In both speed and accuracy, this method is superior to the use of 6-place logarithm tables. My classes pick it up very quickly, and thereafter use it exclusively. One slide rule root and one division on the machine yields results of sufficient accuracy for even such work as transmission line calculation, and one further division gives 10 or 12 place accuracy.

Very truly yours,
L. FUSSELL (A'06, M'22)
(Professor of Elec. Engg., Swarthmore College, Swarthmore, Pa.)

# Standards

## Revisions of Electrical Definitions

For the past 4 months the sectional committee on electrical definitions has been distributing for comment the revisions of the reports of the 17 subcommittees which prepared the "Report on Proposed American Standard Definition of Electrical Terms," issued in printed form by the A.I.E.E. in August 1932. The last of the subcommittee revisions are being prepared for distribution now. After the elapse of the 60-day period permitted for comments on the last distribution the entire revision will then come before the sectional committee for action. In the meantime a new subcommittee has been formed on "electronics." This subcommittee, under the chairmanship of William Wilson (M'23), of the Bell Telephone Laboratories, has as its scope of work the preparation of definitions of tubes, devices, and terms in use in the electronics field and also will attempt to develop an acceptable standard nomenclature for tubes. If successful in this part of their work much of the confusion now existent in literature dealing with electron tubes due to several names being in use for the same type of tube will be cleared up. For further information on electrical definitions work address H. E. Farrer, secretary, sectional committee on electrical definitions, A.I.E.E., 33 West 39th St., New York, N. Y.

## Electrical Insulating Materials

In September 1933 the American Society for Testing Materials issued a book of "Standards for Insulating Materials." This publication, which gives the results of the work of committee D-9 covers in part standard methods of testing molded materials, electrical porcelain, and insulating oils; also tentative methods of testing varnishes, tape, laminated sheet materials, mica, etc. A series of specifications also are given for flexible varnished tubing, tape, rubber gloves, and matting, cotton and silk yarns, asbestos, etc. A copy of the publication may be obtained at a cost of $1.25 by addressing A.S.T.M., 1315 Spruce St., Philadelphia, Pa.

## National Electrical Code

In 1933 edition of the "National Electrical Code" effective November 1, 1933, is now available. The code contains the regulations of the National Board of Fire Underwriters for electric wiring and apparatus as recommended by the National Fire Protection Association. It was approved as American Standard on September 1, 1933, by the American Standards Association. Fully detailed specifications for construction and for performance under test and in service of electrical fittings and materials for use under the regulations of the code are given in the standards of the Underwriters' Laboratories. Address all communications to the National Board of Fire Underwriters, 85 John St., New York, N. Y., or 222 W. Adams St., Chicago, Ill.

# American Engineering Council

## Retaining of Legal Counsel in Washington Discouraged

The practice of engineers and architects retaining lawyers in Washington, D. C., to assist in the securing of contracts from the government as part of its public works program has increased to such an extent that L. W. Robert, Jr. (A'31) assistant secretary of the treasury in charge of public buildings, has requested the American Institute of Architects and American Engineering Council to make widely known the fact that his office seeks to discourage such practice.

Following is the announcement circulated by American Engineering Council:

"The Treasury Department will look with much disfavor on those architects or engineers who retain legal counsel in Washington to aid them in securing professional contracts from the department; in fact, it will be the disposition of the department to eliminate such architects and engineers from consideration altogether.

"Early in the summer the Treasury Department learned that certain Washington lawyers had been soliciting engineers, architects, and others, interested in obtaining Government business, representing that to retain such counsel would enhance the opportunities of the engineers and architects to obtain desirable contracts. This activity has been particularly prevalent in western states.

"The Treasury Department has not made public the names of the lawyers who engaged in this practice, feeling that probably they did not realize: (1) that their proposal was in itself a reflection on certain Government officials; (2) that representation of the nature that lawyers would provide could not possibly have any bearing upon the selections made by the Treasury department.

"The department desires to make its selections on the merits of each case alone. There is no disposition on the part of the department to prosecute any of the parties concerned, but it does want it emphatically understood that such a practice will be outlawed."

# Personal Items

I. M. STEIN (A'18, M'27) director of research, Leeds and Northrup Company, Philadelphia, Pa., has been appointed chairman of the Institute's Sections committee for the year 1933–34. He was born at Long Branch, N. J., and obtained his technical education from the New York Edison evening technical school, Columbia University extension course, and Fort Wayne Correspondence School. In 1911 he joined the meter testing department of the New York (N. Y.) Edison Company, being transferred to the standardizing laboratory of the company in 1912. During part of 1913 he was in charge of instrument rehabilitation after the flood at Dayton, Ohio. During 1914 and 1915 he was engaged on special testing for the New York Edison Company, becoming assistant foreman of the standard laboratory in 1916. During 1917 and 1918 he was foreman of this laboratory and also on a half-time assignment to the chief electrical engineer's office on protective relay work. In 1918 he was senior inspector in the eastern department of the U.S. signal Corps. Shortly after the United States entered the World War, Mr. Stein worked as personal assistant to Thomas A. Edison on the development of submarine and airplane locating devices, and similar apparatus being developed by Mr. Edison in connection with his work on the Naval Advisory Board. Since 1919 he has been with the Leeds and Northrup Company, becoming sales engineer in 1919, in charge of general division sales in 1921, assistant sales manager in 1924, and in 1927 assigned in charge of development, engineering, production, publicity, and sales of automatic combustion control apparatus. He is now director of research for the company. He also is a member of the executive committee of the Leeds and Northrup Company. He has been the author of many technical publications. For the Institute he has been a member of the membership committee 1926–27, 1931–33, and a member of the Sections committee since 1929. He also has served on various committees of the Institute's Philadelphia Section, and has been a member of the board of managers and chairman of this Section. Mr. Stein is a member of the American Physical Society, American Chemical Society, American Electrochemical Society, Illuminating Engineering Society, Franklin Institute, American Society for Testing Materials, American Academy of Political and Social Science, Academy of Political Science, and the American Management Association. He is a member of the Engineers and Art clubs of Philadelphia, and the Old York Road Country Club, Jenkintown, Pa.

I. M. STEIN

M. R. WOODWARD (A'11, M'18) cement sales division, the Babcock and Wilcox Company, Chicago, Ill., has been appointed chairman of the Institute's technical committee on general power applications for the year 1933–34. He was born in Washington, D. C., and was educated at George Washington University, receiving the degree of B.S. in E.E. in 1905, and that of E.E. in 1908. He was for a time draftsman in the motive power department of the Southern Railway Company, and held a similar position in the bureaus of construction and repair ordnance of the U.S. Navy Department. Between 1906 and 1907 he was inspector for the Potomac Electric Power Company. In 1907 he became electrical aid, bureau of yards and docks, U.S. Navy Department, on work covering power plant design and operation and general electrical applications; he remained with this bureau for 10 years. In 1907 he also became instructor in electrical engineering for the National Correspondence Institute, and in 1910 became instructor in electrical engineering at George Washington University, remaining as instructor and assistant professor for many years. At one time he also was head of the mechanical drawing department of the Washington (D. C.) technical high schools. For several years preceding the spring of 1933 he was assistant chief engineer of the Lehigh Portland Cement Company, Allentown, Pa., and now is in the cement sales division of the Babcock and Wilcox Company. He has patents on box making machinery and diving apparatus. Mr. Woodward has been a member of the Institute's committee on general power applications since 1928, and is a past-president of the Engineers' Club of the Lehigh Valley, and is a member of the University Club of Washington, D. C., the Fraternity Club of New York, N. Y., and the Livingston Club of Allentown, Pa.

M. R. WOODWARD

E. B. WAGNER (A'11 M'21) electrical engineer of The Lehigh Valley Coal Company, Wilkes-Barre, Pa., has been appointed chairman of the Institute's technical committee on applications to mining work for the year 1933–34. He was born at Baltimore, Md.; and in 1906 received the degree of mechanical engineer from Cornell University, Ithaca, N. Y. During the summer of 1903, he was in the repair shop of the Baltimore and Ohio Railroad at Baltimore, Md., in the summer of 1904 was with the Maryland Steel Company, Sparrows Point, Md., and in the summer of 1905 was with the C. & P. Telephone Company, Baltimore, Md. Upon graduation in 1906 he returned to the latter company being in the location department and cable testing department. In the latter part of 1906 he joined the signal Department of the Pennsylvania Railroad, Philadelphia, Pa. In 1907 he entered the electrical department of the Lehigh Valley Coal Company, Wilkes-Barre, Pa., being engaged on installation and maintenance of electrical equipment in anthracite mines. In 1911 he became assistant electrical engineer of this company and in 1916 became electrical engineer, which position he now holds. He has served the Institute as a

member of the committee on applications to mining work from 1926 to 1929, and since 1932. He is a member of the Cornell Engineers Society.

E. B. WAGNER

H. C. COLEMAN (A'18, M'28) manager of the marine electrical engineering department of the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa., has been appointed chairman of the Institute's technical committee on applications to marine work for the year 1933–34. He was born at Footville, Ohio; and in 1916 graduated from Ohio State University, Columbus, with the degree of bachelor of electrical engineering. In that year he entered the graduate student course of the Westinghouse Electric and Manufacturing Company at East Pittsburgh, and in 1917 became an electrical engineer in the general engineering division of the company, engaged on application work on electrical apparatus for marine service. His early experience was obtained in the application of electrical equipment to submarines of the United States Navy. He later specialized on diesel-electric ship propulsion and auxiliaries. In 1927 he was appointed manager of marine electrical engineering which position he still holds. Mr. Coleman has assisted in the designing, manufacturing, installing, and testing of ship stabilizers of the gyroscopic type, of submarine and battleship electric propelling plants and auxiliaries, and of diesel-electric and turbine-electric propelling equipment for a large number of ships of all classes. This work has required the spending of considerable time at sea. Mr. Coleman is the author of numerous articles on electric propulsion of ships. Since 1927 he has been a member of the Institute's committee on applications to marine work. He is a member of the Society of Naval Architects and Marine Engineers, the American Society of Naval Engineers, and the Propeller Club of the United States.

A. V. GUILLOU (A'15, M'27) has resigned as chief engineer for the Public Service Commission of Wisconsin, to assume similar duties in the office of the city attorney in the city of Los Angeles, Calif. Mr. Guillou, who was born in Ventura, Calif., graduated in electrical engineering from the University of California in 1912, and then spent 2 years in the apprentice, shop, and test departments of the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa. In 1914 he returned to California as sales engineer for the Pacific Light and Power Corporation, Los Angeles; between 1916 and 1919 was district agent of the Mount Whitney Power and Electric Company, Porterville, Calif. During 1919 and 1920 he was in the engineering department of the Southern California Edison Company at Los Angeles, this company having absorbed the Pacific Light and Power Corporation and the Mount Whitney Power and Electric Company. In 1920 he became assistant engineer of the gas and electric division of the California Railroad Commission, and in 1923 became gas and electric engineer of that commission. There he had supervision of investigations and reports on rates, service, costs, and construction projects of gas and electric utility companies, preparation and presentation of testimony, and preparation of material for written decisions and reports of the commission. In 1926 he became assistant chief engineer of the commission, resigning in

H. C. COLEMAN

1931 to become chief engineer of the Public Service Commission of Wisconsin at Madison, Wis. In the position which he has just assumed in the city attorney's office in Los Angeles, he will handle the engineering phases of the public utility work of that office. Mr. Guillou is the author of an article in this issue.

A. E. KENNELLY (A'88, F'13, past-president) professor emeritus of electrical engineering at Harvard University, Cambridge, Mass., has been elected a member of the International Committee on Weights and Measures. Doctor Kennelly represented the United States at the general conference on weights and measures which was convened at Paris, France, October 3, 1933. This conference is held at intervals of 6 years, and includes representatives from all of the 32 countries which have joined in the Metric Convention.

WILLIAM KELLY (F'25) vice-president and general manager of the Buffalo, Niagara and Eastern Power Corporation, Buffalo, N. Y., was recently elected president of this company. Colonel Kelly has been connected with the company since 1926, when he joined the staff of that utility following 31 years in the United States Army. He was for 5 years prior to 1926 chief engineer of the Federal Power Commission.

F. L. BALL (A'22, M'27) vice-president, Charles H. Tenney & Company, Boston, Mass., and vice-president of a number of public utilities under the management of this company, has also been elected operating vice-president of the Tenney companies in the North Boston Lighting Properties group. All these utilities are now identified with the New England Power Association group.

E. C. CRITTENDEN (A'19, M'22) physicist and chief of the electrical division, U.S. Bureau of Standards, Washington, D. C., has been appointed assistant director of the Bureau of Standards in charge of research and testings. He is president of the Optical Society of America, and a member of the governing board of the American Association for the Advancement of Science.

M. E. LEEDS (A'01, F'26) president of Leeds and Northrup Company, Philadelphia, Pa., has been appointed by Gerard Swope (A'99, F'22) chairman of the business advisory and planning council for the U.S. Department of Commerce to serve as a member of a newly formed committee on unfair trade practices in production and distribution.

E. Y. RICE (A'13) formerly assistant superintendent of lighting for the Hartford Electric Light Company, Hartford, Conn., has been appointed superintendent of the lighting department of this company. He succeeds H. L. Thomson (A'21) who, as announced in ELECTRICAL ENGINEERING for October 1933, p. 729, has been appointed meter and appliance engineer.

L. O. RIPLEY (A'05) for many years vice-president and general manager of the Kansas Gas and Electric Company, Wichita, has recently been appointed president of the company. He has been active in utility development in the Middle West since he became field executive of the Kansas Gas and Electric Company in 1909.

J. F. HANNA (A'08, M'13) formerly president of the Capital Traction Company, Washington, D. C., will be the president and operating head of the Capital Transit Company, formed by the merger of the Capital Traction Company and the Washington Railway and Electric Company. Mr. Hanna also will serve as a director.

WILLIAM MCCLELLAN (A'04, F'12, and past-president) president, Potomac Electric Power Company, Washington, D. C., has been elected a director of the newly formed Capital Transit Company, Washington, D. C., which combines the Capital Traction Company and the Washington Railway and Electric Company.

K. B. THORNTON (A'01, M'13) general manager of the Montreal Tramways Company, Montreal, Quebec, Canada, has been nominated an operating member-at-large of the executive committee of the American Transit Association, for the 3-year term expiring in 1936.

C. B. KEYES (A'03, M'30) district manager, transportation department, General Electric Company, New York, N. Y., has been nominated for manufacturer member-at-large of the executive committee of the American Transit Corporation for the 3-year term expiring in 1936.

L. E. FISCHER (F'29) vice-president of the North American Light and Power Company, Chicago, Ill., has been advanced from the position of executive vice-president of the Northern Natural Gas Company to the position of president of the company.

J. P. HOGAN (M'31) consulting engineer, New York, N. Y., has been elected an "official nominee" of the American Society of Civil Engineers for the position of vice-president, Zone I, of that organization for the term beginning January 17, 1934.

I. E. COX (A'19, M'29) transportation engineering department, General Electric Company, St. Louis, Mo., has been elected secretary of the equipment section of the Midwest Electric Railway Association.

E. P. NELSON (A'27) formerly designing engineer, General Electric Company, Schenectady, N. Y., in the d-c department, is now with the Electric Boat Company, Groton, Conn.

J. W. HICKLIN (A'18) formerly manager of the Richmond, Va., office of the General Electric Company has recently been appointed manager of the Baltimore, Md., office.

H. W. WISCHMEYER (A'09) formerly superintendent of motive power of the Louisville Railway Company, Louisville, Ky., resigned September 30, 1933.

## Obituary

FARLEY OSGOOD (A'05, M'11, F'12, and past-president) consulting engineer, New York, N. Y., died October 6, 1933. He was born at Chelsea, Mass., in 1874. In 1897, he graduated from a 6-year course at Massachusetts Institute of Technology, Cambridge; in 1925 he received the honorary degree of doctor of engineering from Rensselaer Polytechnic Institute. Between 1894 and 1897, he also was in the engineering department of the American Bell Telephone Company, Boston, Mass. Between 1897 and 1898, he was traveling engineer for the New England Telephone and Telegraph Company, covering their territory in Massachusetts, Maine, and New Hampshire, making inspections and installations. In 1898, he went to New Jersey with the New York and New Jersey Telephone Company, acting as chief clerk in the operating department, as special plant engineer, as confident to the vice-president, and as division manager, until 1904. In the latter year he became chief engineer and general manager of the New Milford (Conn.) Power Company. Here he had charge of construction work for the company, and organized its operating forces. In 1908 he joined the organization of the Public Service Electric Corporation of New Jersey as general superintendent; this company now is known as the Public Service Electric Company. In 1917, he became vice-president and general manager of this company, remaining in this position until 1924, when he undertook private practice in New York as a consulting engineer. Doctor Osgood had served the Institute in many capacities, having been manager 1911–14, vice-president 1914–16, and president 1924–25. He was chairman of the New York Section 1921–22, and had been a member of the following committees: Edison Medal, executive, finance, safety codes, standards, coördination of Institute activities, education, meetings and papers (now technical program), and Institute policy. He had also been the Institute's representative on many bodies, which included the electrical committee of the National Fire Protection Association, the national joint committee on overhead and underground line construction, joint power factor committee, Charles A. Coffin fellowship and research fund committee, American Engineering Council, John Fritz Medal board of award, and the U.S. national committee of the International Electrotechnical Commission. Doctor Osgood was a member of the former National Electric Light Association, the New York Electrical Society, and Sigma Chi fraternity. He also was a member of the Engineers and Technology clubs of New York, the Newark (N. J.) Athletic Club, the Orange Lawn Tennis Club of West Orange, N. J., and the Essex County Country Club.

BYRON TURNER BURT (A'96, M'02, F'13 and member for life) vice-president, North American Utilities Corp., Jersey City, N. J., died October 3, 1933. He was born in East Saginaw, Mich., in 1861. He was educated by private tutors and in independent study. In 1883 he accepted a position with the old Edison Isolated Electric Light Company, New York, N. Y. Later the same year he joined the Thomson-Houston Electric Company, where he was in the Lynn, Mass., factory, and on customer installations. In this work he installed many pioneer lighting plants. In 1884 he went to Guatemala, Central America, for the Thomson-Houston Electric Company, returning to the Lynn factory in 1885. After a few months he returned to Guatemala remaining there until the end of 1887, when he was appointed representative and resident engineer for the Thomson-Houston International Electric Company for Italy. In 1889 he went to Bilbao, Spain, for the installation of a large central station. In 1891, he was appointed chief of the lighting engineering department of the Hamburg (Germany) office of the international company, returning to the Boston, Mass., office of the company in 1892. In that year he was appointed special agent of the General Electric Company at the Chicago World's Fair. The same year he was appointed inspector of local plants for the General Electric Company, and later, in 1892, was elected secretary, treasurer, and general manager of the Charleston (S. C.) Light and Power Company, and afterward vice-president of the Charleston Edison Light and Power Company. In 1899 he became secretary, treasurer, and general manager of the Des Moines (Iowa) Edison Light Company. In 1900 he became secretary and superintendent of the Chatanooga (Tenn.) Light and Power Company. Subsequently he became special engineer for the North American Company, New York, N. Y. At the time of his death he was vice-president of the North American Utilities Securities Corporation.

CHARLES FRANCIS CONN (A'12, M'12, F'14) secretary and treasurer of the J. G. White Engineering Corporation, New York N. Y., died October 19, 1933. He was born at Kenosha, Wis., in 1872, and attended 2 years of a 4-year course at Georgia School of Technology, Atlanta. In 1891, he entered the employ of the Thomson-Houston Electric Company (now General Electric Company) as apprentice, later being construction superintendent of the isolated lighting department. In 1895, he engaged in the reconstruction of electric plants and other work in the South, for his own account, and in 1895 became inspector of electrical installation during the construction period, and assistant superintendent of operation during operation, of the electrical power plant for the Cotton States and International Exposition, Atlanta, Ga. In 1896, he again engaged in electrical construction work on his own account. In 1896 he became construction foreman of the electrical department, Flatbush Gas Company, Brooklyn, N. Y. Later in 1898, he became superintendent of the Yonkers (N. Y.) Electric Light and Power Company. In 1899 he returned to the Flatbush Gas Company, becoming superintendent of the electric department of this company in 1900. In 1903 he became special engineering representative of the transformer department of the General Electric Company of Schenectady, N. Y. In 1907 he entered the organization of the J. G. White Engineering Corporation, New York, N. Y., as assistant engineering manager. In 1909 he became assistant manager of the San Francisco office of this company, later becoming manager of this company and remaining in that position until 1916, when he returned to New York, this time as secretary and treasurer of the company, of which he also became a director. Mr. Conn was a member of The American Society of Mechanical Engineers, the American Society of Civil Engineers, and the Engineers and City Midday clubs of New York.

MYLES B. LAMBERT (A'18) transportation sales manager, Westinghouse Electric and Manufacturing Company, New York, N. Y., died September 25, 1933. He was born at Roslyn, N. Y., in 1873. In 1891 he

started work with the Long Island Railroad as brakeman. From brakeman he became a telegraph operator, then ticket agent, train clerk, train dispatcher, and train master, until 5 years from the time he started to work, he had became division superintendent of the Kings County Elevated Railroad. In 1900, realizing the future of electric transportation, he undertook the 2-year apprentice course of the Westinghouse Electric and Manufacturing Company, after which he entered the construction department and was engaged principally in the installation of multiple control apparatus on elevated, subway, and interurban lines, in and near New York City. In 1904 he was transferred to Chicago, Ill., for work on the Chicago Elevated Railroad. In 1907 he returned to the Long Island Railroad to take charge of electrical car equipment, but in 1908 returned to the Westinghouse company as special railway expert. In 1909 he entered the railway sales department as head of the railway equipment division, and in the succeeding years becoming manager of the equipment division, assistant to the manager, and finally manager of the railway department. In 1930 he was made assistant to the vice-president and later transportation sales manager, a position from which he retired a few months ago due to ill health. During a large part of his connection with Westinghouse he was a resident of Pittsburgh. Mr. Lambert was a member of the executive committee of the American Transit Association, the National Council of American Shipbuilders, the Pittsburgh Athletic Association, the Lido Country Club, Long Beach, N. Y., and the Westinghouse Veterans Association.

FREDERICK NEEDHAM BOSSON (F'16) Calumet, Mich., died October 6, 1933. He was born in Boston, Mass., in 1860. Between 1878 and 1880 he studied at Massachusetts Institute of Technology. In 1881 he became engineer for the Ipswich (Mass.) Mills, becoming engineer and designer for a cotton manufacturing company in New Brunswick, Canada. In 1883 he became manager of the Boston office of R. F. Hawkins Iron Works, later that year being engaged on reconstruction of bridges for the Boston and Albany Railroad for the Hawkins Iron Works. Between 1884 and 1886 he remained with the Hawkins company, engaged on work for the Boston and Albany Railroad and the Massachusetts Central Railroad. Between 1886 and 1891, he was engaged in the Chicago, Ill., office of the Thomson-Houston Electric Company. In 1891 he joined the organization of the Calumet and Hecla Mining Company, remaining with this organization in various capacities for 40 years. He was electrician between 1891 and 1898, and after 1898 was electrical engineer and consulting engineer on hydraulic and mechanical engineering for the Calumet and Hecla Mining Company and its subdivisions. He also had served as consulting engineer on mining electrical work for other organizations, and as confidential engineer for 2 trust companies for bond issues. He was a member of the Miscowabig Club, Calumet.

WILLIAM AUGUSTINE FERGUSON (A'04) recently with the Union Electric Light and Power Company, St. Louis, Mo., died September 10, 1933. He was born at Ferguson, Mo., in 1871. His general education was carried through the preparatory department of Marietta (Ohio) College, and his technical studies were kept up for 6 years thereafter at night. He then spent 2 years in the Westinghouse Electric and Manufacturing Company's factory at East Pittsburgh, Pa., the last 6 months of which he was in charge of the commercial test of a-c and d-c generators. Four years were then spent on the road, inspecting light, power and railway plants; the last year of these 4 he was on the Pacific Coast on transmission line work. In 1901 he went to Mexico City, Mexico, to reconstruct the substation of the local hydroelectric company, resigning 2 years later on account of sickness. In 1903, he became connected with the Mexican Light and Power Company of Mexico City and Montreal, as electrical engineer in charge of construction work in Mexico City and vicinity. Subsequently he spent many years in New York City, Mexico, Cuba, and South America, connected with the Electric Bond and Share Company. Since February 1933, he had been associated with the Union Electric Light and Power Company.

JOHN CONKLIN BENJAMIN (A'18) Electrical Research Products, Inc., Hollywood, Calif., died September 21, 1933. He was borne at Greenport, L. I., N. Y., in 1893. He received his education from Friends Academy, Locust Valley, N. Y., and at the New York Electrical School. In 1912 he entered the employ of the Westinghouse Electric and Manufacturing Company, Newark, N. J., as meter tester, and soon thereafter was transferred to the standard room. Later the same year he was employed by the Splitdorf Electric Company, Newark, N. J., on the construction and repair of high and low voltage magnetos. In 1913 he joined the Westinghouse Lamp Company, being in the experimental department until 1914 when he was transferred to the engineering department. In 1915 he joined the H. W. McCandless Company, as outside salesman, being transferred to the sales organization of the Westinghouse Lamp Company as traveling salesman later in the same year. In 1917 he entered the employ of the Western Electric Company, New York, N. Y., as engineer, subsequently being transferred to the Electrical Research Products, Inc., a subsidiary of the Western Electric Company, at Hollywood.

VANCE W. MILLER (A'32) chief engineer, Texas Electric Service Company, Fort Worth, Texas, died September 12, 1933. He was born in Springtown, Texas, in 1893, and graduated from the Agricultural and Mechanical College of Texas, in 1914, with the degree of B.S. in E.E. During the latter half of 1914 he was with the Texas City Transportation Company, as steam-electric station operator. Between 1914 and 1916, he was switchboard operator for the Brush Electric Company, Galveston, Texas, and for the following year was meter tester of the Fort Worth (Texas) Power and Light Company. From 1917 to 1925 he was superintendent of the meter department for this company, and between 1925 and 1929 was distribution engineer. In 1929 he was placed in charge of engineering for the successor company, the Texas Electric Service Company. Mr. Miller was a past-president of the Fort Worth Electric Club and of the Fort Worth A. and M. Club.

EDSON L. MORRIS (A'30) sales engineer, Westinghouse Electric and Manufacturing Company, Salt Lake City, Utah, died October 6, 1933. He was born in Lewiston, Idaho, in 1905. In 1927 he graduated from the University of Idaho with the degree of B.S. in E.E. He thereupon joined the organization of the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa., being transferred to the Salt Lake City office in 1929. In that city he had taken an active part in civic and fraternal circles. For the Institute he was at the time of his death chairman of the Utah Section.

# Membership

## Applications for Election

Applications have been received at headquarters from the following candidates for election to membership in the Institute. If the applicant has applied for direct admission to a grade higher than Associate, the grade follows immediately after the name. Any member objecting to the election of any of these candidates should so inform the national secretary before November 30, 1933 or January 31, 1934, if the applicant resides outside of the United States or Canada.

Anderson, G. H., Westchester Ltg. Co., Ossining, N. Y.
Austin, S. P. (Member), Am. Tel. & Tel. Co., Atlanta, Ga.
Ayo, E. R., Am. Tel. & Tel. Co., Atlanta, Ga.
Binetruy, J., 245 Palisade Ave., Bridgeport, Conn.
Bower, H. F., Gen. Cable Corp., Rome, N. Y.
Cantley, J. V., United Shoe Machy. Corp., Beverly, Mass.
Crawford, W. P., U.S. Forest Service, Coeur d'Alene, Idaho.
Crider, F. B., Gen. Elec. Co., Washington, D. C.
Cuneo, F. N., Westchester Ltg. Co., Ossining, N. Y.
Davis, J. W., Southern Bell Tel. & Tel. Co., Atlanta, Ga.
Dean, R. (Member), Southern Bell Tel. & Tel. Co., Atlanta, Ga.
Doonan, J. J. (Member), Southern Bell Tel. & Tel. Co., Atlanta, Ga.
Engelken, R. C. (Member), Kliegl Bros., Universal Elec. Stage Ltg. Co. Inc., N. Y. City.
Glover, R. P., Am. Tel. & Tel. Co., Atlanta, Ga.
Goss, E. K., Indiana Bell Tel. Co., Indianapolis
Harris, H. C., Am. Tel. & Tel. Co., Atlanta, Ga.
Hebner, N. E. (Member), Prest-O-Lite Storage Battery Corp., Indianapolis, Ind.
Keith, J. M., Am. Tel. & Tel. Co., Atlanta, Ga.
Lang, E. C., 1472 Myrtle Ave., Bklyn., N. Y.
Lynch, W. J., Am. Tel. & Tel. Co., Atlanta, Ga.
Malsbary, J. S., Wagner Elec. Corp., St. Louis, Mo.
Maltby, A. R., Standard Oil Co. of Indiana, Sugar Creek, Mo.
Mathewson, E. P. Jr., N. J. Bell Tel. Co., Newark N. J.
Miller, R. McI., Am. Tel. & Tel. Co., Atlanta, Ga.
Miroddi, S., 1115 73rd St., Bklyn., N. Y.
Petelle, J. J., J. J. Petelle Elec. Co., Fair Haven, Vt.
Rogers, J. H., Eastman Kodak Co., N. Y. City.

Shawver, J. W., Okla. Gas & Elec. Co., Oklahoma City.
Terry, A. L., Am. Tel. & Tel. Co., Atlanta, Ga.
Waldhausen, M. H. (Member), Siemens, Inc., N. Y. City.
Wascheck, G., Am. Tel. & Tel. Co., N. Y. City.
Wolf, C. D., Wolf Eng. Co., Indianapolis, Ind.
32 Domestic

Foreign
Gross, E., A. E. G. Union Elektrizitats-Gesellschaft, Vienna, Austria.
Marshall, H. F., Shanghai Pwr. Co., Shanghai, China.
Marsouk, I. Y., Metro-Vickers Elec. Co., Trafford Park, Manchester, 17, Eng.
Whyte, C. L. A., 166 Bath Rd., Southsea, Hants, Eng.
4 Foreign

## Recommended for Transfer

The board of examiners, at its meeting of September 28, 1933, recommended the transfer of the following members to the grades of membership indicated. Any objection to these transfers should be filed at once with the national secretary.

To Grade of Fellow

Eldredge, Mark, chief engr., Memphis Pwr. & Lt. Co., Memphis, Tenn.
Groeneveld-Meijer, American representative, Allgemeine Elektricitaets-Gesellschaft, Schenectady, N. Y.
Kartak, Franz A., dean of engg., Marquette Univ., Milwaukee, Wis.
Waugh, Lester R., consulting business, New York.

To Grade of Member

Boveri, Theodore, dir. of traction dept., Brown Boveri & Co. Ltd., Baden, Switzerland.
Burke, Charles T., engg. mgr., Gen. Radio Co., Cambridge, Mass.
Burke, George E., dist. supt., Pub. Serv. Elec. & Gas Co., Passaic, N. J.
Campbell, Richard D., engr., Am. Tel. & Tel. Co., New York.
Coop, Edward R., distribution engr., So. County Pub. Serv. Co., Westerly, R. I.
Cullwick, Ernest G., asst. prof. of E.E., Univ. of British Columbia, Vancouver, B. C., Can.
Doolittle, Fred B., radio engr., Southern Calif. Edison Co., Los Angeles, Calif.
Euler, Wm. G. B., gen. supt., East Bay & San Fran. Divs., Pacific Gas & Elec. Co., San Francisco, Calif.
Fouraker, Raymond S., prof. of E.E., No. Car. State College, Raleigh.
Harcus, Wilmore C., sound recording supervisor, United Artists Studio Corp. Ltd., Hollywood, Calif.
Mead, Fred B., gen. engr., Westinghouse E. & M. Co., Syracuse, N. Y.
Nystrom, C. W., gen. outside plant engr., Southwestern Bell Tel. Co., St. Louis, Mo.
Quinn, George E., test engr., N. Y. Edison Co., New York.
Rollow, J. Grady, chief elec. engr., Los Angeles Gas & Elec. Co., Los Angeles, Calif.
Snyder, Edwin H., engr., Pub. Serv. Elec. & Gas Co., Newark, N. J.
Thomson, Charles J., erecting engr., Contract Serv. Dept., Gen. Elec. Co., Schenectady, N. Y.

## Addresses Wanted

A list of members whose mail has been returned by the postal authorities is given below, with the address as it now appears on the Institute record. Any member knowing of corrections to these addresses will kindly communicate them at once to the office of the secretary at 33 West 39th St., New York, N. Y.

Blackhall, Harold J., Postlagernd, Essen, Germany.
Boicourt, Frank R., Rockwell City, Iowa.
Bugnion, Frank E., 14 Clinton St., Cambridge, Mass.
Deming, Paul S., Oklahoma Gas & Elec. Co., Seminole, Okla.
Eberhard, 2191 Plaza, Schenectady, N. Y.
Endicott, E. M., 2020 Monroe St., Toledo, Ohio.
Ghamat, S. B., School of Engg. of Mil., Milwaukee, Wis.
Hamby, H. M., 708 F St., N. E., Washington, D. C.
Hirsch, Chas. J., Level Club Hotel, 253 W. 73rd St., N. Y. City.
Kantayya, A. G., 38 Rue Eurest Charles, Marcinelle, Belgium.
Lober, Charles, K. C. P. & L. Co., 1330 Baltimore Ave., Kansas City, Mo.
McDonnell, John D., Box 691, Burlington, N. C.
Mowat, George, 230 Mather St., Oakland, Calif.
Strommer, 7229 Penn Ave., Pittsburgh, Pa.

# Engineering Literature

## New Books in the Societies Library

Among the new books received at the Engineering Societies Library, New York, during September are the following which have been selected because of their possible interest to the electrical engineer. Unless otherwise specified, books listed have been presented gratis by the publishers. The Institute assumes no responsibility for statements made in the following outlines, information for which is taken from the preface or text of the book in question.

APPLICATION of the CATHODE RAY OSCILLOGRAPH in RADIO RESEARCH. By R. A. Watson Watt, J. F. Herd and L. H. Bainbridge-Bell. London, Dept. of Sci. & Ind. Res.; obtainable from British Library of Information, New York, 1933. 290 p., illus., 10x6 in., cloth, $2.55. Gives detailed information about the design and construction of the apparatus developed at the British Radio Research Station, for the application of the cathode ray oscillograph to the problems of radio research, and is intended for others engaged in work along similar lines. The book aims to be an introductory, practical handbook upon the technique of frequency investigation with the oscillograph.

BUSINESS under the RECOVERY ACT. By L. Valenstein and E. B. Weiss. N. Y. & London, McGraw-Hill Book Co., 1933. 314 p., 8x5 in., cloth, $2.50. The question here considered is the effect which the National Recovery Act will have upon the passage of goods from the manufacturer to the ultimate consumer. The book discusses merchandizing, selling, and advertising under the new conditions, with special consideration of ultimate, rather than immediate effects.

(The) GYROSCOPE, its Laws and Mysteries. By T. Bodde. Tientsin-Peiping (China), Peiyang Press, 1933. 23 p., diagrs., 10x7 in., cloth, $1.00. This very brief non-mathematical explanation of the laws of the gyroscope comes from Peiyang University, and is intended to provide an understanding of the principles of gyroscopic action which will enable readers to follow practical applications with understanding.

HEROIC AGE of SCIENCE, the Conception, Ideals, and Methods of Science among the Ancient Greeks. By W. A. Heidel. Publ. for Carnegie Inst. of Wash. by Williams & Wilkins Co., Baltimore, 1933. 203 p., 9x6 in., cloth, $2.50. Exhibits by examples the way in which the Greeks thought of science and tried to solve its problems, and points out the relation between their procedure and the processes of the mind in practical affairs.

INDUSTRIAL ELECTRICAL MEASURING INSTRUMENTS. By K. Edgcumbe and F. E. J. Ockenden. London & N. Y., Isaac Pitman & Sons, 1933. 583 p., illus., 9x6 in., cloth, $7.50. A treatise upon the design, construction, and use of the many forms of instruments now available, written for the practical engineer. A comprehensive account of modern types of instruments for all purposes is given, with discussions of design and construction. This edition is practically a new work.

INTERNAL-COMBUSTION ENGINES, Theory and Design. By V. L. Maleev. N. Y. & London, McGraw-Hill Book Co., 1933. 386 p., illus., 9x6 in., cloth, $4.00. The principles involved in the design and operation of internal-combustion engines are presented in combination with detailed instruction in methods of designing. The book is intended for use as a college text and by engineers and designers.

LABOR RELATIONS under the RECOVERY ACT. By O. Tead and H. C. Metcalf. N. Y. & London, McGraw-Hill Book Co., 1933. 250 p., 8x5 in., cloth, $2.00. Aims to supply practical guidance to those interested in methods of organized dealing with employees in industries which are under the provisions of the National Recovery Act. The possibilities of employee representation and of company unions and labor unions are discussed in the light of the experience of the authors.

ÖFFENTLICHE HEIZKRAFTWERKE und ELEKTRIZITÄTSWIRTSCHAFT in STÄDTEN. By E. Schulz. Berlin, Julius Springer, 1933. 209 p., illus., 10x7 in., 28.50 rm. The possibilities and advantages of combining district heating and electricity supply are set forth in detail. The principles of heat and power distribution are discussed, existing district heating systems in Europe and America are described and the design of new systems is considered.

PLANNING for the SMALL AMERICAN CITY (Publication 32). By R. V. Black. Chicago, Public Administration Service, 1933. 90 p., illus., 11x8 in., paper, $1.00. This monograph considers planning in cities with a population under 50,000, from the point of view of operating officials and interested citizens. How a plan may be made, what planning offers to a city, and how to carry out a plan are discussed.

REINFORCED CONCRETE-MECHANICS and DESIGN. By R. A. Caughey. Ann Arbor, Mich., Edwards Bros., 1933. 115 p., illus., 11x8 in., paper, $2.70. Aims to assist the undergraduate student. Early chapters discuss fundamentals, the web stresses in beams, the design of beams and columns, and combined bending and direct stress. Chapters are devoted to moments in beams and frames, to floors carrying concentrated loads, and to composite members of concrete and structural steel. Other chapters deal with the design of buildings, arches and rigid frames, dams, and retaining walls.

SCHIFFBAU (Ausgewählte Schweisskonstruktionen, Bd. 5). By Lottmann. Berlin, VDI-Verlag, 1933. 50 p., illus., 12x8 in., cloth, 9 rm. A collection of 50 plates of photographs and drawings illustrating applications of welding to shipbuilding. The examples are chosen from the practice at the principal German shipyards. Brief descriptive notes in English accompany each plate.

TELLING the WORLD. (Century of Progress Series). By G. O. Squier. N. Y. & London, Century Co., 1933. 163 p., illus., 8x5 in., cloth, $1.00. The story of the development of electrical communication, from the electrical experiments of William Gilbert to the radio telephone of today; is told in brief, but readable fashion.

TWENTY-FIVE YEARS of CHEMICAL ENGINEERING PROGRESS, 1908–1933. Silver Anniversary Volume, American Institute of Chemical Engineers. Edit. by S. D. Kirkpatrick. D. Van Nostrand Co., N. Y., 1933. 373 p., illus., 9x6 in., cloth, $4.00. Twenty-five essays by prominent specialists describe the advances during the last quarter century in the principal industries served by the chemical engineer. The industries include the manufacture of chemicals, paper, sugar, soap, paints, plastics and glass, the processing of coal and refining of petroleum, fractional distillation, evaporation, water purification, etc.

ANALYTIC and VECTOR MECHANICS. By H. W. Edwards. N. Y. & Lond., McGraw-Hill Book Co., 1933. 428 p., illus., 9x6 in., $4.00. Intended for students of mathematics and physics. A thorough knowledge of calculus and some training in college physics are prerequisites. The range of topics is limited to those that are fundamental. Vector methods are used freely, frequently parallel with the analytical treatment.

AIR CONDITIONING. By J. A. Moyer and R. U. Fittz. N. Y. & Lond., McGraw-Hill Book Co., 1933. 390 p., illus., tables, 9x6 in., cloth, $4.00. Presents theory of air cooling and discusses air filtering, cooling methods, refrigeration, humidity control, fans, the design of plants for office buildings, theaters, restaurants, factories, cars, and residences. Design requirements are considered and the necessary computations are given for various structures.

ALTERNATING CURRENT CIRCUITS. By M. P. Weinbach. N. Y., Macmillan Co., 1933. 417 p., illus., 9x6 in., cloth, $4.50. This textbook has been designed to meet the increasing need for a fuller understanding of circuit theory and more complete knowledge of methods of analysis and schemes of solution. A rational treatment of circuit theory is presented, suited to the needs of undergraduate students of electrical engineering.


**Engineering Societies Library**

29 West 39th Street, New York, N.Y.

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Resources of the library are available also to those unable to visit it in person. Lists of references, copies or translation of articles, and similar assistance may be obtained upon written application, subject only to charges sufficient to cover the cost of the work required.

A collection of modern technical books is available to any member residing in North America at a rental rate of five cents per day per volume, plus transportation charges.

Many other services are obtainable and an inquiry to the director of the library will bring information concerning them.

# Industrial Notes

**G-E Reports Increased Business.**—The General Electric Company has added 7600 employes to its payrolls since March 1, and the total annual payroll rate is today $17,000,000 greater than it was on that date, Gerard Swope, president, made known October 24 in a statement to the company's 187,000 stockholders mailed with regular dividend checks. New business booked the first nine months of the year has shown a steady rise, and for the first time since 1929 orders have exceeded those for a like period of the previous year. Another significant fact is that this year, for the first time in three years, orders for the third quarter totaled more than the sales billed in the same period. In referring to the N.R.A., President Swope declared General Electric has conformed not only to the letter but to the spirit of the Act and began operating under the Electrical Manufacturers Code, as to hours of employment and wages, a week before the code went into effect August 15.

**Recent Westinghouse Orders.**—Electrical equipment totaling more than $1,000,000 for the six 1500-ton U.S.N. destroyers being built in private shipyards has been ordered from the Westinghouse Elec. & Mfg. Co. The equipment includes condensers, air ejectors, forced draft blowers, pumps, switchboards, and other electrical equipment. Recent steel mill orders include cold roll strip motors and control equipment for the Inland Steel Co., at Indiana Harbor, Ind., approximating $60,000; motors and control equipment for the Youngstown Sheet & Tube Co., Indiana Harbor, over $60,000; motors and control apparatus for American Sheet & Tin Plate Co., at Gary, Ind., about $75,000. The Detroit Edison Company has ordered motors and control equipment to cost $60,000 for improvements now being made at the Connors Creek power station.

**Large Trolley Equipment Order to G-E.**—As part of a general street railway improvement program, costing $2,550,000, the Indianapolis Railways recently announced the purchase of 80 new trackless trolley cars and 50 new street cars. Contracts signed with the J. G. Brill Co. and the General Electric Co. provide for early delivery. The new street cars will each be equipped with 4 G-E motors, control and air-brake compressors and the trolley coaches with 2 G-E motors, control and air-brake compressors. The first step of the modernization program, initiated about a year ago by the railway company, was the purchase of 25 modern street cars, 15 trolley coaches and 30 gas busses.

**New Underground Conduit.**—The Goodyear Tire & Rubber Co. has announced a new type of conduit for underground electrical service. The new product is to be marketed exclusively by the Graybar Electric Co. Advantages claimed for the conduit are its high resistance to water absorption, definite fire resistance, unusually high impact and crushing strength. In tests at extremely high temperatures the conduit does not exude oils or impregnating compounds. The inside surface is smooth and glossy so that cable pulling strains are greatly reduced. No rubber is used in the manufacture of the conduit but it is composed of high quality cotton fibres and specially developed compounds, none of which will cause deterioration of rubber covered or weatherproof wire, lead sheaths, copper or steel. The conduit does not deteriorate when exposed to the elements. It is furnished in units from 5 to 8 feet in length, with inside diameters from 1 to 6 inches.

**New Industrial Tester.**—A new portable a-c testing set, known as the "Industrial Analyzer," suitable for testing household electrical appliances and industrial apparatus and motors up to 100 hp., 440 volt, is announced by the Westinghouse Elec. & Mfg. Co. Capable of making complete tests up to 125 amperes, 500 volts, alternating current, the analyzer is in reality, a miniature portable three-phase switchboard panel in a carrying case complete with all necessary switching equipment, transformers, and resistors. The unit contains a triple scale ammeter 0–5, 0–25, and 0–125 amperes, a triple range voltmeter 0–150, 0–300, and 0–600 volts, a polyphase wattmeter with three scales for the above voltage and current ranges, and a polyphase power factor meter with 10–100–80 scale to operate on the above voltages and currents. The instruments are of a newly designed miniature type with standard size mechanism having high overload capacity and low energy consumption. The scales of the instruments are $3\frac{1}{2}$ inches long. Two special lightweight current transformers with hipernik cores and with primary winding tapped for 5, 25, and 125 amperes are connected into the circuit in the conventional three-phase wiring scheme to provide the multiple current ranges. The various voltage ranges are obtained by switching tapped resistors in the potential circuits.

# Trade Literature

**Instruments.**—Bulletin GEA-1784, 8 pp. Describes new, medium-size portable instruments, Type AP-9—voltmeters, milliammeters, ammeters, and wattmeters. While designed especially for a-c measurement, they will also serve satisfactorily in many applications for d-c measurements. General Electric Co., Schenectady, N. Y.

**Stiffness Tester.**—Bulletin E-10133, 4 pp. Describes a new precision stiffness and resiliency tester for measuring the stiffness and elasticity of flexible materials such as paper, cloth, thin metals, etc. Smith-Taber, North Tonawanda, N. Y.

**Commutators and Slip-Rings.**—Bulletin. Describes a manufacturing and repair service for all types and sizes of commutators and slip-rings. It includes both the refilling of existing shells and the manufacture of complete new assemblies. Columbia Electric Mfg. Co., 1292 East 53rd St., Cleveland, Ohio.

**Weatherproof Wire Specifications.**—Bulletin, 12 pp. Contains the Utilities Research Commission Specifications for weatherproof wires and cables. Tables are included giving the weights of the various types of such conductors. General Cable Co., 420 Lexington Ave., New York.

**Electric Water Heaters.**—Catalog 282-A. Describes various types and sizes of electric water heaters to meet every central station load condition, including storage tank heaters, strap-on heaters and heaters of the side-arm circulation and immersion types. Westinghouse Elec. & Mfg. Co., Mansfield, O.

**Identified Cord Movement.**—Bulletin, 16 pp. Outlines the complete story of the identified cord movement participated in by the principal manufacturers of flexible cords for electrical appliances. As a result, less than 25% of the cord now made is unapproved, whereas a year ago only 25% was approved. Electrical Cord Manufacturers, NEMA, 155 East 44th St., New York.

**Hollow Cable.**—Bulletin, 20 pp. Describes Type HH, single layer, hollow core copper, overhead transmission cable. This conductor is, in effect, an economical and flexible tube which is made in the long lengths required. The conductor, itself, is a single layer which is self-supporting, no internal supporting members being required. Due to the fact that the individual segments are tightly interlocked, the conductor is very strong, maintains a circular cross section and is practically uncrushable, even between flat surfaces. The Type HH hollow conductor has the following characteristics and advantages, according to its makers: The enlarged diameter required to prevent corona can be obtained without exceeding the cross section needed to carry the current; adequate flexibility—when the cable is bent the segments, which are twisted together in a spiral, slide on each other; simplicity; lowest corona losses for given diameter; high resistance to vibration; mechanical stability; electrical resistance—all of the metal in the conductor is used to its fullest efficiency; flashover characteristics—an arc violent enough to destroy one or two outer wires of ordinary cable will do no more than to create a burr on the strong type HH segments. Also, if a segment is burned through, it will be held in place by the adjacent segments and will not unravel. Many miles of this type of conductor have been in successful European operation, most of it for 5 years. It will be used exclusively on the transmission lines from Boulder Dam to Los Angeles, 2 circuits each, 271 miles long (1,626 conductor miles) to operate at 290,000 volts. General Cable Co., 420 Lexington Ave., New York.